

# Recitation 1

## Chapter 12

**Problem 12.2.** In an engine, a piston oscillates with simple harmonic motion so that its position varies according to the expression

$$x = (5.00 \text{ cm}) \cos(2t + \pi/6) \quad (1)$$

where  $x$  is in centimeters and  $t$  is in seconds. At  $t = 0$ , find (a) the position of the piston, (b) its velocity, and (c) its acceleration. (d) Find the period and amplitude of the motion.

(a) Simply plugging in  $t = 0$  into the given position equation we have

$$x(t = 0) = 5.00 \text{ cm} \cdot \cos(\pi/6) = 4.33 \text{ cm} \quad (2)$$

(b) Then we take the derivative to get the velocity

$$v = \frac{\partial x}{\partial t} = -2 \text{ s}^{-1} \cdot 5.00 \text{ cm} \cdot \sin(2t + \pi/6) \quad (3)$$

$$v(t = 0) = -10 \text{ cm/s} \cdot \sin(\pi/6) = -5.00 \text{ cm/s} \quad (4)$$

(c) Taking the derivative again yields the acceleration

$$a = \frac{\partial v}{\partial t} = -2 \text{ s}^{-1} \cdot 10.00 \text{ cm/s} \cdot \cos(2t + \pi/6) \quad (5)$$

$$a(t = 0) = -20 \text{ cm/s}^2 \cdot \cos(\pi/6) = -17.3 \text{ cm/s}^2 \quad (6)$$

(d) The period is the time it takes for a complete revolution, i.e. for a phase change of  $2\pi$ , so  $T = \pi \text{ s}$ . The amplitude is the maximum distance from equilibrium. Looking at the original equation for  $x(t)$ , we realize that  $|\cos(\theta)| \leq 1$  for all  $\theta$ , so the biggest  $x$  will ever get is  $A = 5.00 \text{ cm}$ .

**Problem 12.5.** A particle moving along the  $x$  axis in simple harmonic motion starts from its equilibrium position, the origin, at  $t = 0$  and moves to the right. The amplitude of its motion is  $2.00 \text{ cm}$  and the frequency is  $1.50 \text{ Hz}$ . (a) show that the position of the particle is given by

$$x = (2.00 \text{ cm}) \sin(3.00\pi t) \quad (7)$$

Determine (b) the maximum speed and the earliest time ( $t > 0$ ) at which the particle has this speed, (c) the maximum acceleration and the earliest time ( $t > 0$ ) at which the particle has this acceleration, and (d) the total distance traveled between  $t = 0$  and  $t = 1.00 \text{ s}$ .

(a) Because  $x(t = 0) = 0$ , we can express the motion

$$x(t) = A \sin(\omega t) . \quad (8)$$

(this is Equation 12.6 with  $\phi = -\pi/2$ , because  $\cos(\theta - \pi/2) = \sin(\theta)$ .)

To find  $A$ , note that  $\sin(\theta)$  increases as  $\theta$  increases from 0, so the particle's initially rightward motion requires  $A > 0$ .  $|\sin(\theta)| \leq 1$  so  $|x| \leq A$ , and the amplitude is given as  $2.00 \text{ cm}$  so  $A = 2.00 \text{ cm}$ .

To find  $\omega$ , simply compute

$$\omega = 2\pi f = 2\pi \cdot 1.50 \text{ Hz} = 3\pi \text{ rad/s} . \quad (9)$$

Plugging our  $A$  and  $\omega$  into our  $x(t)$  yields the equation of motion we set out to find.

(b) To find the maximum speed, we could either take the derivative of  $x(t)$  (like we did in 12.2), or realize that the derivative will have another factor of  $\omega$  in it's amplitude and jump to the answer  $v_{\text{max}} = A\omega = 6\pi \text{ cm/s}$ .

The maximum speed occurs when the position is zero. Our particle starts at  $x = 0$ , so it has maximum speed at  $t = 0, T/2, T, 3T/2, \dots$ . We're asked for the first occurrence for  $t > 0$ , so  $t = T/2 = 1/2f = 0.333 \text{ s}$

(c) To find the maximum acceleration, we could either take the derivative of  $v(t)$  (like we did in 12.2), or realize that the derivative will have another factor of  $\omega$  in it's amplitude compared to the velocity and jump to the answer  $a_{\text{max}} = A\omega^2 = 18\pi^2 \text{ cm/s}^2$ .

The maximum acceleration occurs at the minimum position, because  $F = ma = -kx$ . Our particle starts at  $x = 0$ , so it at minimum extension at  $t = 3T/4, 7T/4, \dots$ . We're asked for the first occurrence for  $t > 0$ , so  $t = 3T/4 = 3/4f = 0.500 \text{ s}$

Note that in (b) we were looking for the maximum scalar *speed*, so the direction didn't matter, but in (c) we were looking for the maximum vector *acceleration*, so the direction did matter.

(d) The period of our particle is  $T = 1/f = 2/3$  s.  $t = 1.00$  s  $= 1.5T$ . That means it travels  $0 \rightarrow A \rightarrow -A \rightarrow A \rightarrow 0$ , for a grand total of  $d = A + 2A + 2A + A = 6A = 12.0$  cm.

**Problem 12.12.** A 1.00 kg glider attached to a spring with a force constant of 25.0 N/m oscillates on a horizontal, frictionless air track. At  $t = 0$ , the glider is released from rest at  $x = -3.00$  cm (that is, the spring is compressed by 3.00 cm). Find (a) the period of its motion, (b) the maximum values of its speed and acceleration, and (c) the position, velocity, and acceleration as functions of time.

(a)

$$\omega = \sqrt{\frac{k}{m}} = 5.00 \text{ N/m} , \quad (10)$$

and

$$T = \frac{1}{f} = \frac{2\pi}{\omega} = 1.26 \text{ s} . \quad (11)$$

(b) The maximum speed of the object is (see 12.5)

$$v_{\max} = \omega A = 15.0 \text{ cm/s} , \quad (12)$$

where  $A = 3.00$  cm.

The maximum acceleration of the object is (see 12.5)

$$a_{\max} = \omega^2 A = 75.0 \text{ cm/s}^2 . \quad (13)$$

(c) The position starts at the minimum value of  $x$ , so a  $-\cos$  based expression for  $x$  is in order

$$x = -A \cos(\omega t) = -A \cos(\omega t) \quad (14)$$

$$v = \omega A \sin(\omega t) = v_{\max} \sin(\omega t) \quad (15)$$

$$a = \omega^2 A \cos(\omega t) = a_{\max} \cos(\omega t) . \quad (16)$$

**Problem 12.15.** A block of unknown mass is attached to a spring with a spring constant of 6.50 N/m and undergoes simple harmonic motion with an amplitude of 10.0 cm. When the block is halfway between its equilibrium position and the end point, its speed is measured to be 30.0 cm/s. Calculate (a) the mass of the block, (b) the period of the motion, and (c) the maximum acceleration of the block.

(a) Conserving energy

$$E = \frac{1}{2}kA^2 = 32.5 \text{ mJ} \quad (17)$$

$$E = \frac{1}{2}k\left(\frac{A}{2}\right)^2 + \frac{1}{2}mv^2 = \frac{E}{4} + \frac{1}{2}mv^2 \quad (18)$$

$$m = \frac{2}{v^2} \cdot \frac{3}{4}E = \frac{3E}{2v^2} = 542 \text{ g} . \quad (19)$$

(b)

$$T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} = 1.81 \text{ s} . \quad (20)$$

(c) This is just Hooke's law

$$F = ma_{\max} = kA \quad (21)$$

$$a = \frac{k}{m}A = 1.20 \text{ m/s}^2 . \quad (22)$$

**Problem 12.18.** A 2.00 kg object is attached to a spring and placed on a horizontal, smooth surface. A horizontal force of 20.0 N is required to hold the object at rest when it is pulled 0.200 m from its equilibrium position (the origin of the  $x$  axis). The object is now released from rest with an initial position of  $x_i = 0.200$  m, and it subsequently undergoes simple harmonic oscillations. Find (a) the force constant of the spring, (b) the frequency of oscillations, and (c) the maximum speed of the object. Where does this maximum speed occur? (d) Find the maximum acceleration of the object. Where does it occur? (e) Find the total energy of the oscillating system. Find (f) the speed and (g) the acceleration of the object when its position is equal to one third of the maximum value.

(a) The external force  $F_e$  must exactly balance the spring force  $F_s$  to hold the object at rest on a frictionless surface, so

$$|F_s| = k|x| = |F_e| \quad (23)$$

$$k = \left| \frac{F_e}{x} \right| = 100 \text{ N/m} . \quad (24)$$

(b) The frequency of oscillation is then

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = 1.13 \text{ Hz} , \quad (25)$$

and

$$\omega = 2\pi f = 7.07 \text{ rad/s} . \quad (26)$$

(c) The maximum speed of the object is (see 12.5)

$$v_{\max} = \omega A = 7.07 \text{ rad/s} \cdot 0.200 \text{ m} = 1.41 \text{ m/s} , \quad (27)$$

which occurs at  $x = 0$ , when all the oscillation energy is kinetic.

(d) The maximum acceleration of the object is (see 12.5)

$$a_{\max} = \omega^2 A = (7.07 \text{ rad/s})^2 \cdot 0.200 \text{ m} = 10 \text{ m/s}^2 , \quad (28)$$

which occurs at  $x = -A = -0.200$  m. If you are only interested in the peaks in the *magnitude* of the acceleration, they occur for  $x = \pm A$ .

(e) Lets find the energy in the initial situation, right after the object was released. It's at rest, so its kinetic energy is 0 and all the energy is spring-potential energy

$$E = \frac{1}{2} k A^2 = 2.00 \text{ J} \quad (29)$$

(f) Conserving energy

$$E = \frac{1}{2} k \left( \frac{A}{3} \right)^2 + \frac{1}{2} m v^2 = \frac{E}{9} + \frac{1}{2} m v^2 \quad (30)$$

$$v = \pm \sqrt{\frac{2}{m} \cdot \frac{8}{9} E} = \pm 1.33 \text{ m/s} . \quad (31)$$

(g) This is very similar to Problem 12.15 (c).

$$F = ma = kx \quad (32)$$

$$a = \frac{k}{m} x = \pm \frac{kA}{3m} = \pm 3.33 \text{ m/s}^2 . \quad (33)$$