PHYS 325: Computational Physics III

Winter 2023

Homework #6
(Due: March 17, 2023)

1. (a) Solve the diffusion equation using the Crank-Nicolson scheme on a grid of `J` = 1001 points, with a time step of `dt` = 0.1. Start with the fundamental solution at time $t = 1$ and integrate the system forward in time to $t = 5$. Plot your solution at $t = 5$ and compare it to the fundamental solution at that time.

   (b) Plot the maximum absolute difference between the numerical and analytic solutions as a function of time for $1 \leq t \leq 5$, for time steps `dt` = 0.1, 0.5, and 1.0.

2. (a) The temperature T of a uniform medium is governed by the diffusion equation with diffusion coefficient D. The medium extends from $x = 0$ to $x = L$. Initially the medium is at temperature $T = 0$, and then the temperature of the left edge ($x = 0$) is raised to $T[0] = 1$ and held at that value. Use the Crank-Nicolson scheme with $J = 1001$ points to determine the time taken for the right edge ($x = L$) to reach a temperature of $T[J-1] = 0.5$ for (i) $D = 1, L = 10$, (ii) $D = 4, L = 10$, and (iii) $D = 1, L = 5$. Do these times agree with our earlier estimates of the time scale for diffusion across the grid?

   (b) Now the initial state of the medium is as in part (a), but the temperature of the left edge varies in time as

   $$T[0] \;=\; \cos^2 t.$$

   What is the steady-state temperature of the right edge, for $D = 1, L = 10$?

3. Exercise 10.1.

4. Exercise 10.2.