PHYS 325: Computational Physics III

Winter 2023

Homework #5
(Due: March 3, 2023)

1. Using the `wave1d.py` script, create two sine waves at the ends of the grid, one of wavelength 2 and amplitude 1 in $-10 < x < -6$, moving to the right, the other of wavelength 2 and amplitude 0.5 in $6 < x < 10$, moving to the left. Use `J` $= 501$ zones in the simulation and impose absorbing boundary conditions. Note that a pure sine wave will have at least one of `u`, `r`, or `s` discontinuous at $t = 0$, which (as we saw in class) causes serious problems when integrating a second-order differential equation. To avoid this, force all quantities to go smoothly to zero at the ends of the ranges, as in the "smoothly truncated wave pulse" (option 4) in `wave1d.py`. As the waves meet they interfere. Determine the maximum value of the field `u` on the grid during the simulation, and plot $u(x,t)$ at the instant when this maximum occurs. What do you expect this maximum value to be, from elementary physics? Does your numerical result agree with your expectations?

2. (a) Create a virtual "wave tank" by modifying the 2-D wave solver `wave2d.py` so that the left-hand side ($x = -10$, for all y) oscillates in time with

```
u = np.sin(np.pi*t),   rx = 0,   ry = 0,   s = du/dt.
```

   Use a 251×251 grid for your calculation, with an absorbing boundary at the right ($x = 10$), and run to time $t = 20$. Reduce boundary effects by forcing the y derivatives to be zero at the top and bottom edges:

```
u[:,0] = u[:,1]
u[:,J-1] = u[:,J-2]
```

   and similarly for the other variables. You should see a nearly plane wave propagating across the domain. Plot $u(x, 0, t)$ as a function of x at the end of the simulation and hence measure the wavelength. What wavelength do you expect, from elementary physics? Does your numerical result agree with your expectations?

   (b) What happens if you put an absorbing barrier in the tank at $x = 0$ by setting

```
u = rx = ry = s = 0
```

   at $x = 0, -2 < y < 2$? Create an image of the field $u(x, y, t)$ at the end of the simulation, $t = 20$.

3. Now instead of a plane wave, modify the left-hand boundary condition in `wave2d.py` to represent two "point" sources of amplitude 5 and frequency 1, each of height 0.25 units, centered on $y = 2.5$ and $y = -2.5$. Create an image of the field $u(x, y, t)$ at the end of the simulation, $t = 20$. Does what you see agree with your expectations based on elementary physics? Specifically, how would you expect the resultant interference pattern to change as you vary the frequency and the distance between the sources? (Try doubling and halving each.)