PHYS 325: Computational Physics III

Winter 2023

Homework #3

(Due: February 10, 2023)

A unit charge is at rest at the origin. At time $t = 0$ it starts to execute (relativistic) simple harmonic motion along the x-axis, with

$$X(t) = 0.5 \sin t\,, \qquad Y(t) = 0\,.$$

Units are such that $c = 1$.

1. According to the discussion in class, the fields due to the particle at any point $\mathbf{r}$ depend on the *retarded time* $t_{\rm ret}(\mathbf{r}, t)$, defined implicitly by

   $$t_{\rm ret} = t - |\mathbf{r} - \mathbf{R}(\mathbf{t}_{\rm ret})|/c\,,$$

   where $\mathbf{R} = (X, Y)$. Plot $t_{\rm ret}$ as a function of time for $-10 < t < 20$ at the points (i) $\mathbf{r} = (5, 0)$ and (ii) $\mathbf{r} = (0, 2)$.

2. Plot the x and y components of the electric field at the point $\mathbf{r} = (5, 5)$ as functions of time, for $-10 < t < 20$.

3. Plot the *radial* and *transverse* components of the electric field at time $t = 10$ as functions of $r = |\mathbf{r}|$, for $1 < r < 20$, along the lines (i) $x = 0, y > 0$ and (ii) $y = x, x > 0$.

4. Plot the electric field lines (in the $x - y$ plane) due to the charge. Draw 16 field lines (uniformly spaced in angle) starting on the circle $x^2 + y^2 = 1$, and show the field within the box $-20 < x < 20$, $-20 < y < 20$, at observation times (i) $t = 1$, (ii) $t = 5$, and (iii) $t = 10$. (Use a step size $\Delta s = 0.05$ along the field lines.)

   Note: Apart from the fact that computing the field now involves determining the retarded time $t_{\rm ret}$, and the expression for the field is now (much) more complicated than previously, the program you wrote for Homework #1 should be usable more or less as is.