PHYS 325: Computational Physics III

Winter 2023

Homework #2

(Due: February 3, 2023)

1. Use Monte-Carlo integration to estimate the volume and mass contained within a sphere of radius 1 ($x^2 + y^2 + z^2 < 1$) that also lies below the plane $x + y + z = \frac{1}{2}$ (i.e. $x + y + z < \frac{1}{2}$) and above the plane $x + y + 2z = 0$. The density is $\rho(x, y, z) = e^{-xyz}$. Perform your calculations with N random points in the volume under consideration, for $N = 100, 10000$, and 1000000. Turn in your program and the results of the integration (i.e. your estimate of the volume and mass), along with an error estimate, for each value of N.

2. A stratified medium is uniform in the x-direction and has a refractive index that varies with height (y in the terminology of Exercise 4.1) as
$$n(y) = 1 + 0.5y^4.$$
Find the light path of least time between the points $(0, 1)$ and $(1, 1)$, using a grid of 21 points uniformly spaced in x and starting from a path with y_i randomly distributed between 0 and 1. What is the minimum value of y along this path?

3. Following the discussion in class, write a program to determine the equilibrium distribution (in 2 dimensions) of N identical charges initially randomly distributed within a conductor. The shape of the conductor is defined by the ellipse
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\,,$$
where we will take $a = 1\,\mathrm{m}$, $b = 0.25\,\mathrm{m}$. Start by distributing charges randomly and uniformly within the 2×2 square containing the ellipse, accepting only those charges that lie within the ellipse. Continue until N charges have been placed inside the conductor. For definiteness, take each charge to be $1\mu C$.

   Use the Monte-Carlo technique discussed in class to find the equilibrium configuration of the charges. Don't forget to use the proper 2-dimensional form of the potential, $\phi(r) = -kq \ln r$. You should find that the charges distribute themselves on the surface of the conductor. Using $N = 1000$ charges, print out and turn in the initial and final configurations of the system, along with your program. Based on your graph, estimate the number of charges per unit length (just count charges within small arc lengths of the ellipse) near the points $(a, 0)$ and $(0, b)$. How is this charge density related to the curvature of the surface?

4. Repeat the calculation of the previous problem, with the following changes:

   (i) Use a circular conductor (i.e. take $a = b = 1\,\mathrm{m}$).

   (ii) Assume that an external field of magnitude $E = 10^7\,\mathrm{Vm}^{-1}$ acts in the positive x-direction. Include the contribution from this field in the total potential.

   (a) As before, plot the initial and final configurations of the system. How does the presence of the external field affect the final charge distribution?

   (b) Plot the field lines of the final configuration. What should the field be inside the conductor? What value do you find numerically at the center?