Fundamental Solution of the Diffusion Equation

One very important standard solution to the diffusion equation

$$\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial x^2}$$

has initial conditions

$$u(x,0) = \delta(x)\,.$$

The time-dependence if this **fundamental solution** is

$$u(x,t) = \frac{1}{\sqrt{4\pi Dt}}\; e^{-x^2/4Dt}\,,$$

a result which is easily verified by direct substitution.