Computation of Electric Field Lines II

Some Practical Considerations

Our program will proceed as follows.

1. For each charge q_i in the system, compute the field lines starting on a small circle centered on that charge. The radius of the circle is most conveniently set by requiring that the magnitude of the potential due to q_i on the circle have some standard value $\phi_0 > 0$. The starting points for the field lines around charge i are thus defined by

$$\begin{aligned} X_j &= x_i + r\cos\theta_j\,, \\ Y_j &= y_i + r\sin\theta_j\,, \qquad \text{for } j = 1, \ldots, n_\theta \end{aligned}$$

   where $r = k|q_i|/\phi_0$ and $\theta_j = 2j\pi/n_\theta$. In practice, choosing $n_\theta = 8$ or 16 will give a sufficient number of field lines. In order to avoid the termination condition preventing the calculation from even starting, it is advisable to start with r a little bigger than the above value—$r = 1.1k|q_i|/\phi_0$, say.

   If we choose standard units such that the "typical" value of X is 1 unit (i.e. 1 meter) and typical charges are 1 μC, taking $\phi_0 = 10^6\,\mathrm{V}$ gives reasonable results.

2. Field lines are computed by choosing a step size δs and taking a step. For the standard units just described, it is sufficient to take $\delta s = 0.01$ (1 centimeter) in most cases. However, close to a charge, where the field is varying rapidly, it is desirable to take shorter steps. We do this by placing a limit $\delta\phi_{\max}$ on the change in potential over the distance δs. Since $|\delta\phi| \approx |\mathbf{E}|\delta s$, this requires $\delta s < \delta\phi_{\max}/|\mathbf{E}|$. Our choice of δs is thus

$$\delta s = \min(0.01,\ \delta\phi_{\max}/|\mathbf{E}|)\,.$$

   As a practical matter, in standard units, we take $\delta\phi_{\max} = 10^2\,\mathrm{V}$.

3. Finally, we must specify the circumstances under which we stop the computation of the field line and move on to the next. The simplest criteria are that we stop if $|\mathbf{X}| \equiv \sqrt{X^2 + Y^2} > R_{\max}$ (field line too far from the origin) or if $|\phi(\mathbf{X})| < \phi_0$ (too close to another charge). We take $R_{\max} = 50$.

In constructing this program, you will find it helpful to write, in turn, functions to

1. initialize the charges and their positions
2. compute the potential at any point due to the charges
3. compute the electric field at any point due to the charges
4. determine the field line starting at some specified point
5. loop over all starting points around each positive charge and compute the field line through each.