PHYS 325: Computational Physics III

Winter 2023

Exercise 6.1

1. Write a program to find the numerical solution $u(x,t)$ of the 1-dimensional advection equation

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial x} = 0\,,$$

where v is constant, for $-10 \le x \le 10$, subject to the initial condition

$$\begin{aligned} u(x,0) &= \sin(\pi x) \qquad (-2 \le x \le 0) \\ u(x,0) &= 0 \qquad\qquad (\text{otherwise})\,, \end{aligned}$$

and boundary conditions $u(-10,t) = u(10,t) = 0$, using a simple FTCS differencing scheme:

$$u_j^{n+1} = u_j^n - \frac{v\Delta t}{2\Delta x}(u_{j+1}^n - u_{j-1}^n)\,.$$

Take $v = 1$ and perform the calculation on a grid of 201 points spanning the range $-10 \le x \le 10$ (so $\Delta x = 0.1$), with time step $\Delta t = 0.1$. How does your numerical solution compare with the analytical solution at times $t = 0.5$, $t = 1$, $t = 2$, and $t = 5$?