PHYS 325: Computational Physics III

Winter 2023

Exercise 4.2

1. Distribute a set of N identical positive charges uniformly over the interior of the circle

$$x^2 + y^2 = 1\,.$$

Start by distributing the charges randomly and uniformly within the 2×2 square containing the circle, and accept only those charges that lie within the circle. For definiteness, take each charge to be $1\mu C$.

Use a Monte-Carlo technique to find the equilibrium configuration:

(a) Choose a charge at random.

(b) Randomly change both its x and y coordinates by amounts in the range $(-\delta, \delta)$, where we will take $\delta = 0.01$ here. If the charge would move outside the circle, constrain it to end up precisely *on* the circle.

(c) Accept the above change if and only if its effect is to *reduce* the total potential energy of the system. Otherwise, restore the charge's old coordinates. In fact, calculating the total potential energy of the system is unnecessary (in addition to being very expensive), because if we move just one charge (i, say), then the contributions to the total potential energy from all the other pairs of charges are unchanged. Only the contribution from the potential at i changes. Thus we need only check that the potential at charge i decreases. Operationally, we choose i, calculate the potential at (x_i, y_i), move i, recalculate the potential at the new location, and reject the change if the potential has not decreased. This will make a big difference to the speed of your script!

(d) Repeat from (a) above. Stop when N attempted changes in a row are rejected.

You should find that the charges distribute themselves on the surface of the conductor. Using $N = 1000$ charges, plot the initial and final configurations of the system.