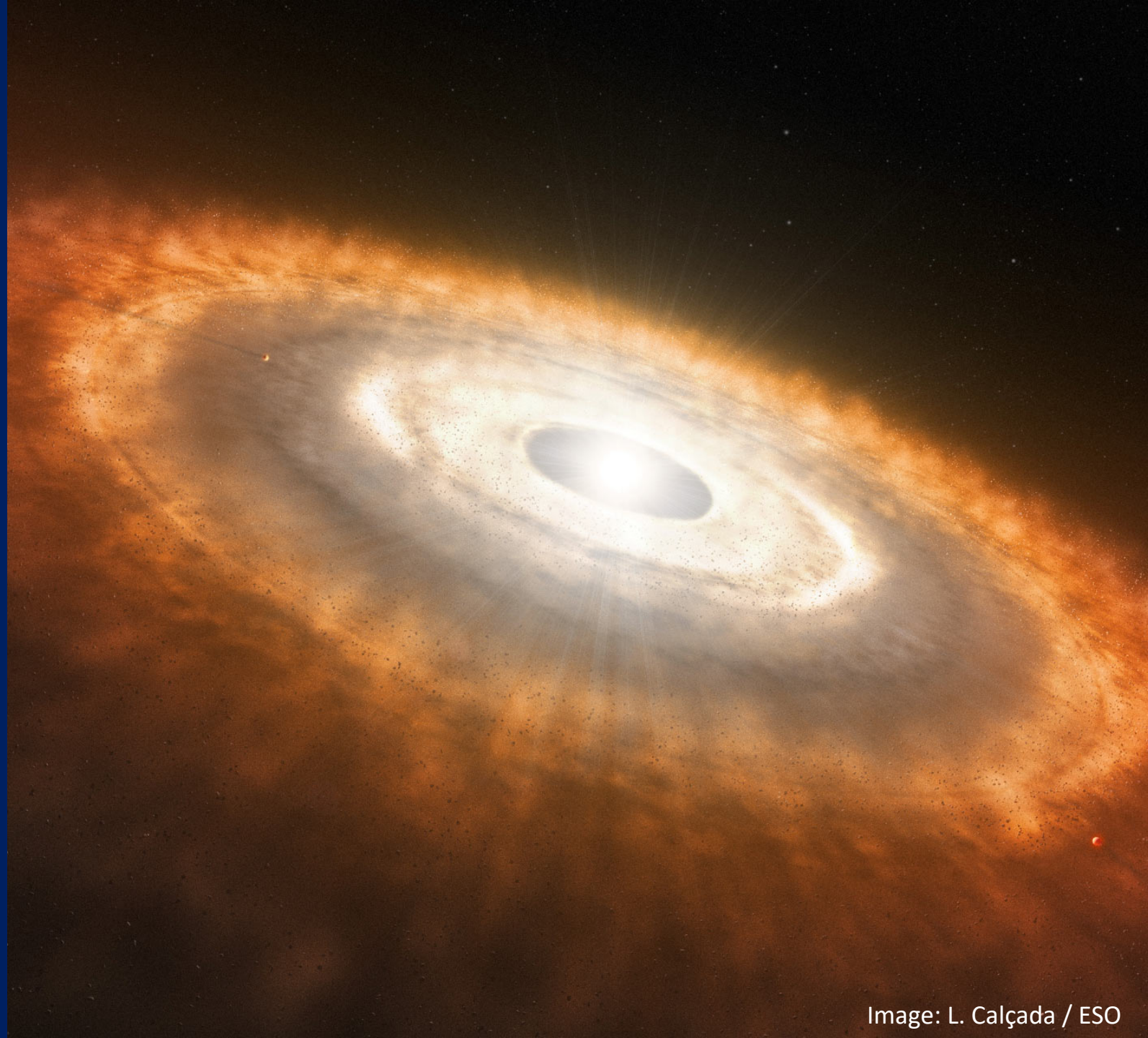


USING FLASH TO
CREATE A
PROTOPLANETARY
DISK

Sean Lewis

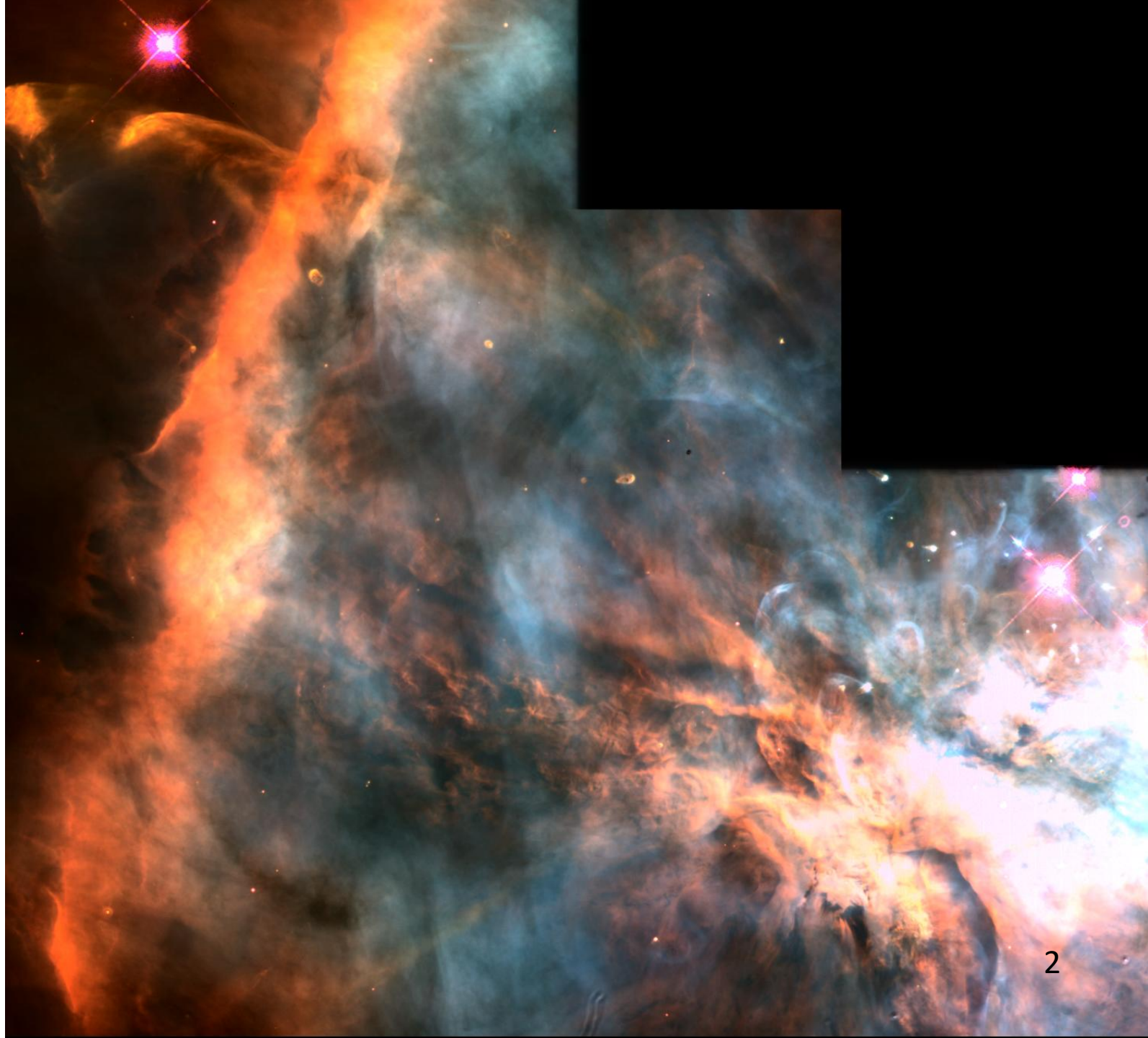
Drexel University

June 5, 2019



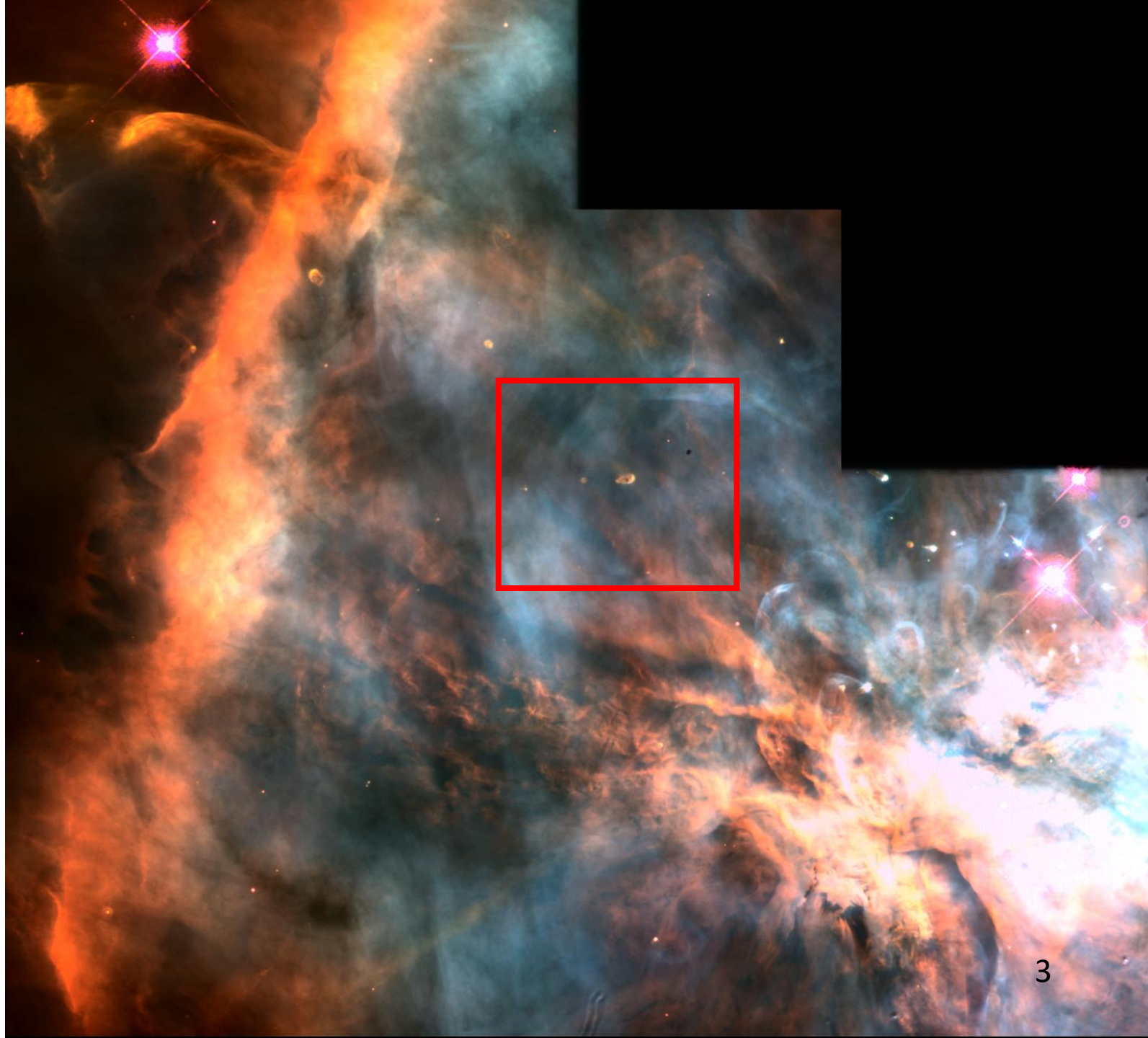
What are protoplanetary disks?

- Gaseous remnants around stars born out of collapsed stellar nebulae.
- Precursor to our Solar System.
- May be crucial to the chemical composition of the Solar System.



What do Protoplanetary Disks Look Like?

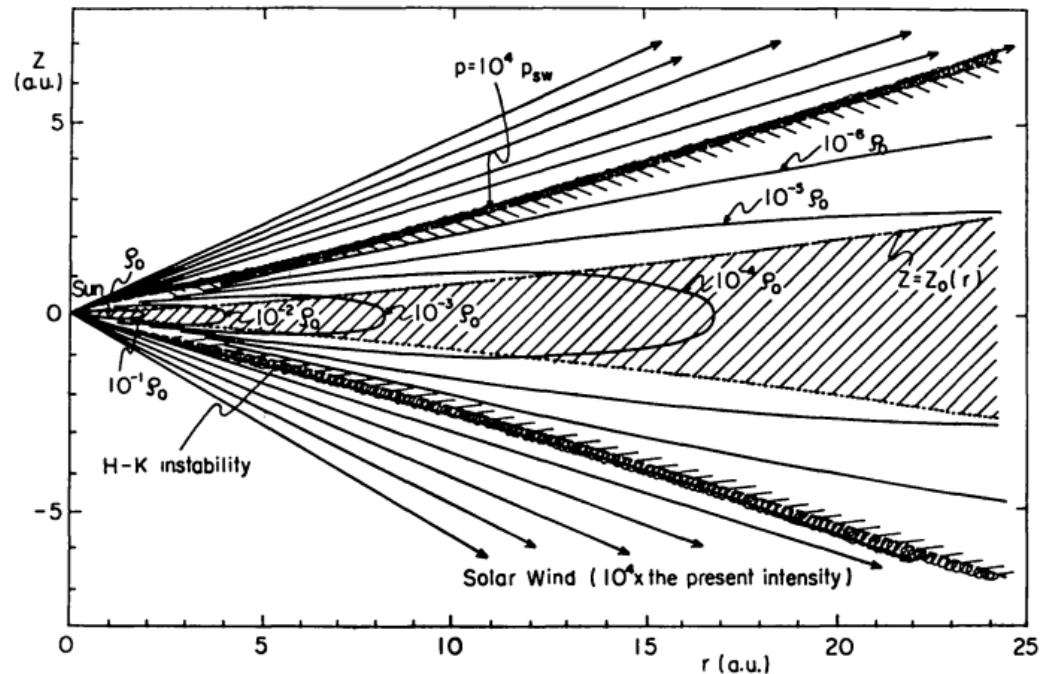
- Match the mass distribution of our Solar System. (Lynden, B., Pringle J. 1974; Hayashi, C. 1985).
- “Nice” model to reconcile planetary formation. (Desch 2007).
- O’dell 1994 image of Orion proplyd disks embedded in star cluster.





Mathematical Models

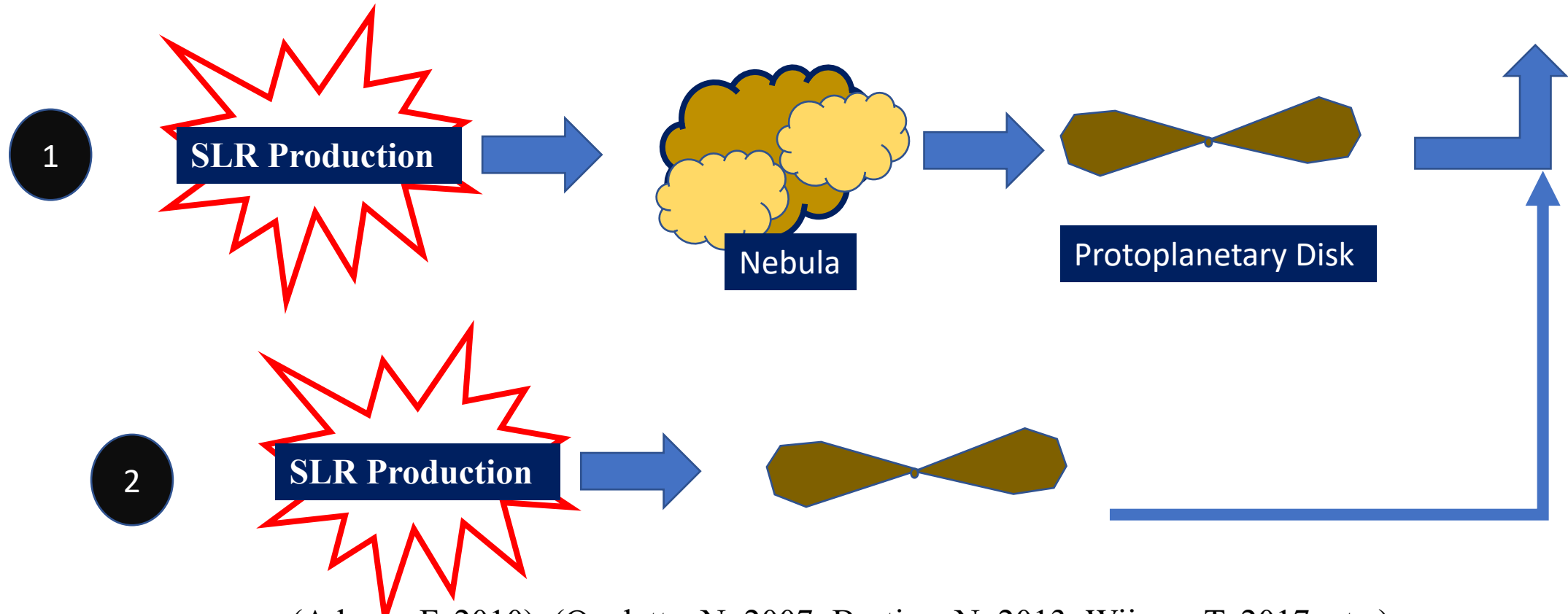
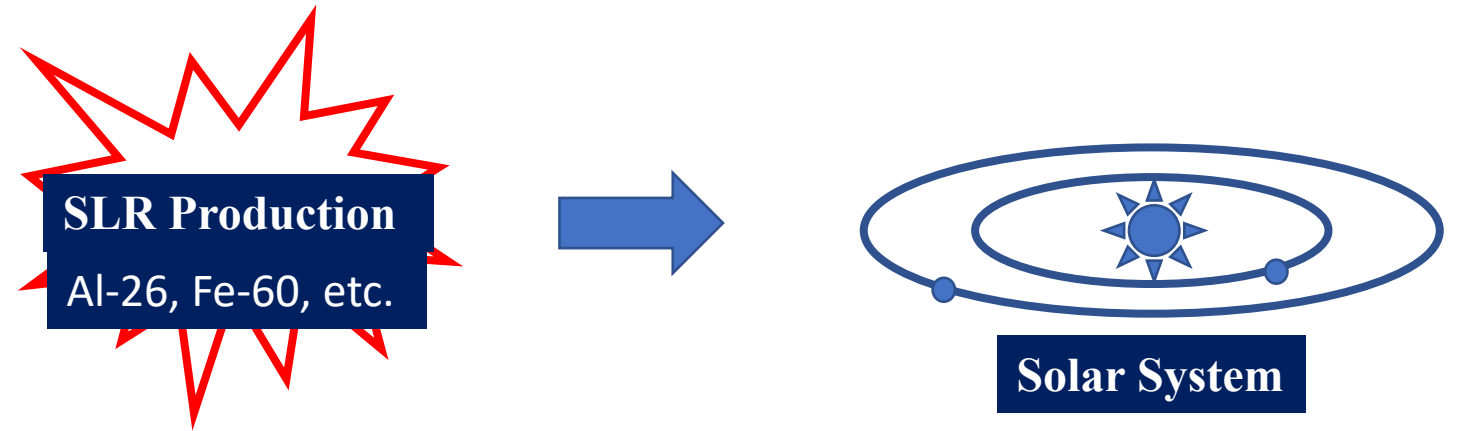
- Various radial density distributions.
- Hydrostatic equilibrium.



$$\rho(r, z) = \rho_0 r^{-2.75} \exp\{-z^2 / z_0^2(r)\}$$

Hayashi, C. (1985)

Short-Lived Radioisotopes in the Solar System



(Adams, F. 2010), (Ouelette, N. 2007; Bastian, N. 2013; Wijnen, T. 2017; etc.)

My Goal

Computational Framework

Explore SLR injection, disk
survivability

Control

Initial conditions of disk
formation

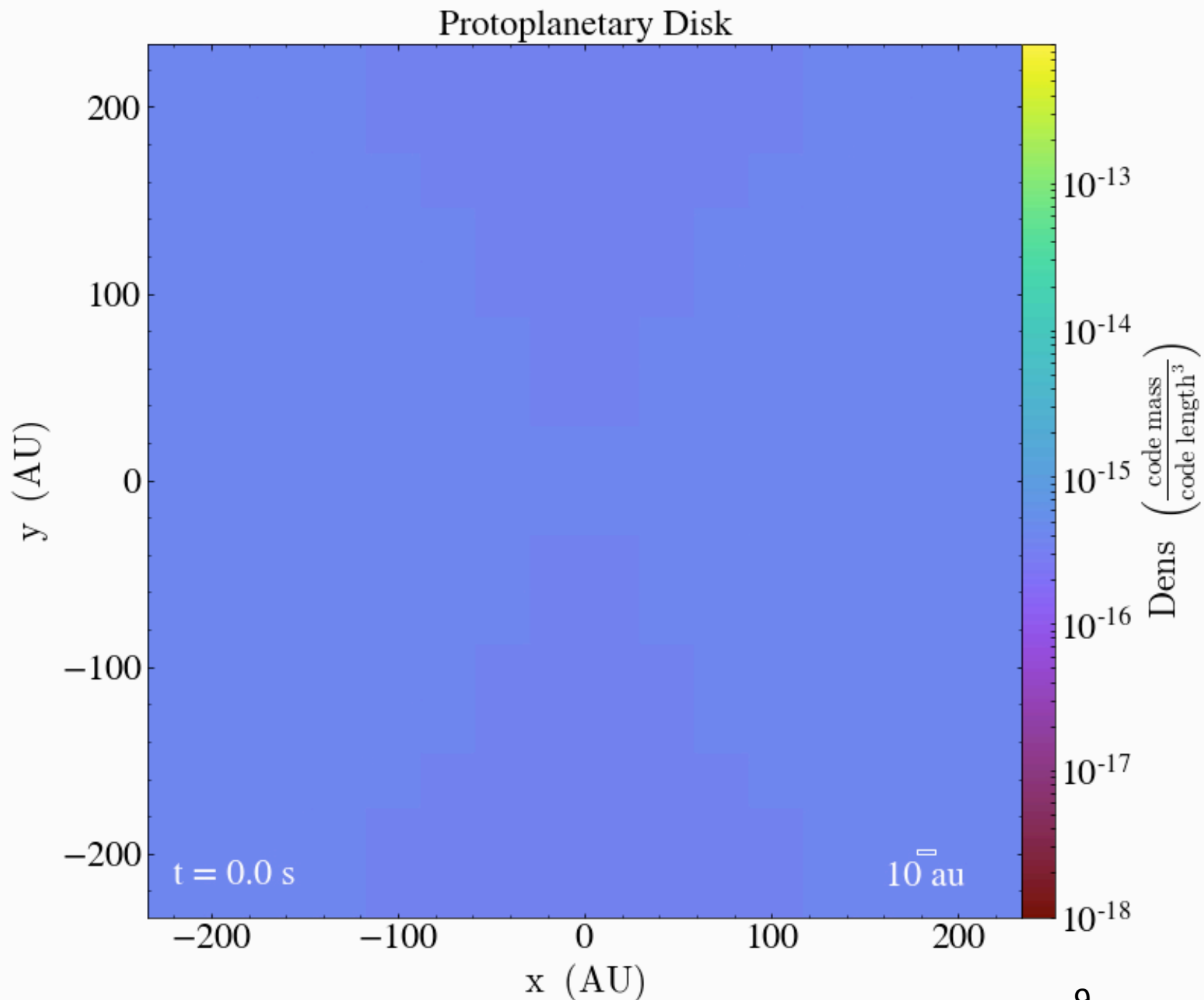
Hydrodynamics

FLASH

(Fryxell et al. 2000)

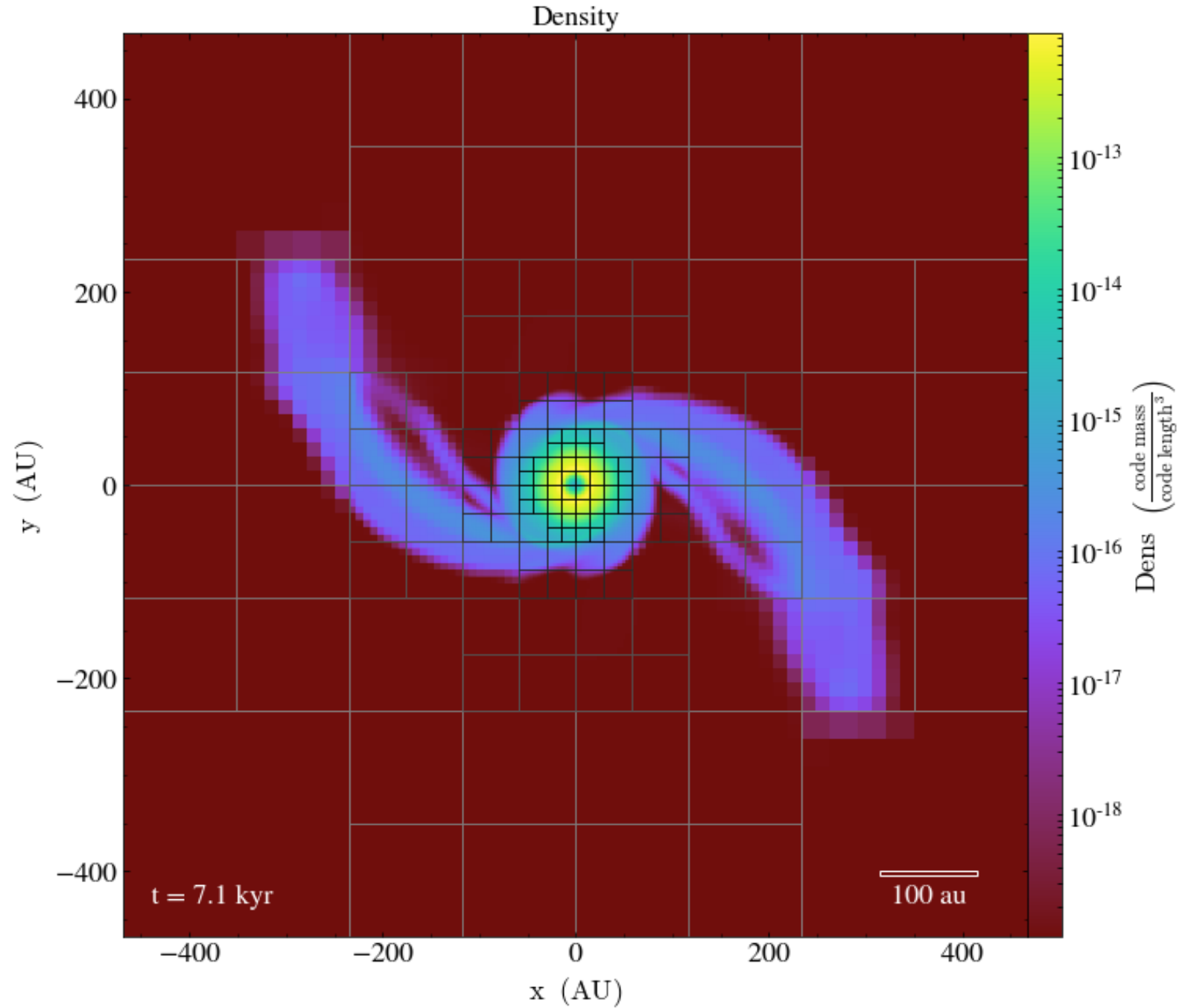
- Hydrodynamics
- Self-gravity
- Sink Particles
- Adaptive Mesh Refinement (AMR)

Visualizations created using **yt**
software package (Turk et al. 2011)

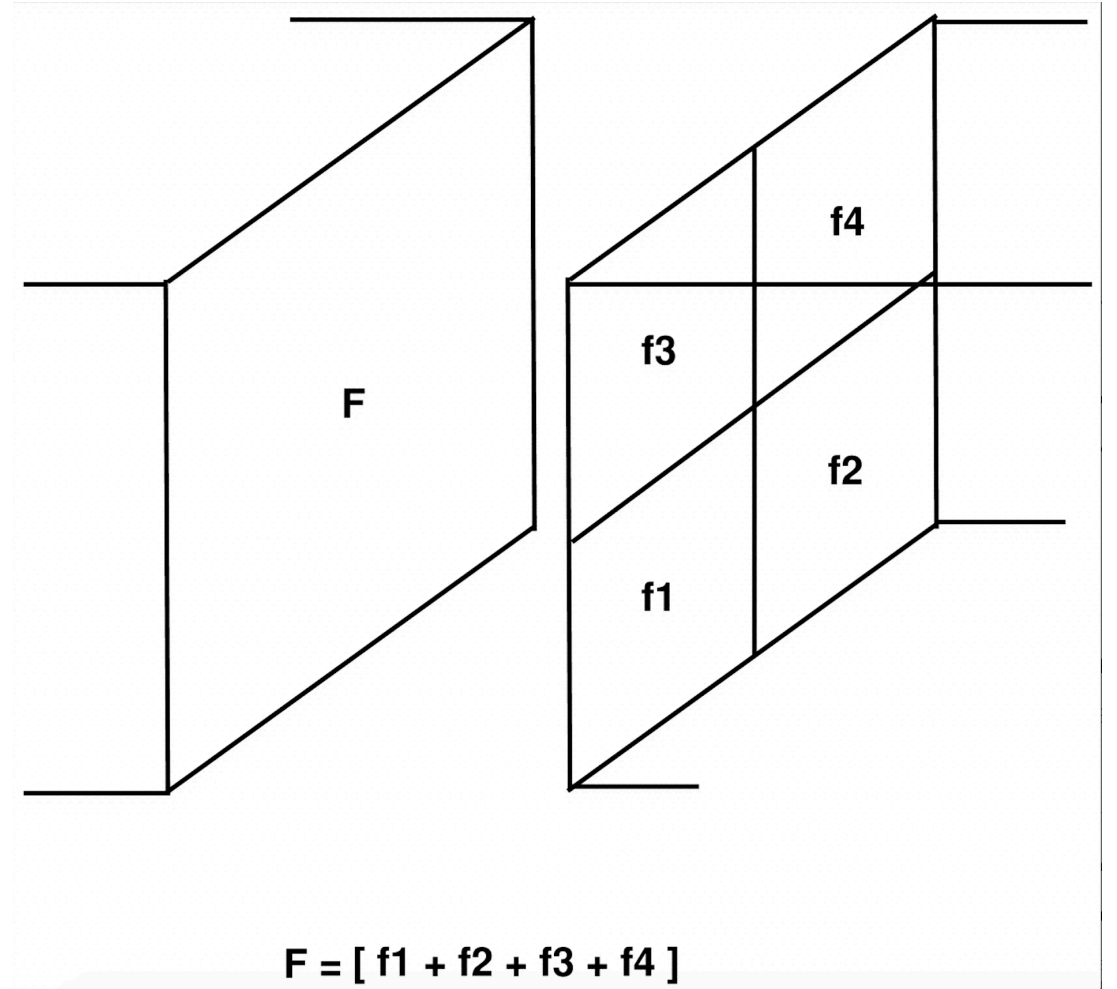
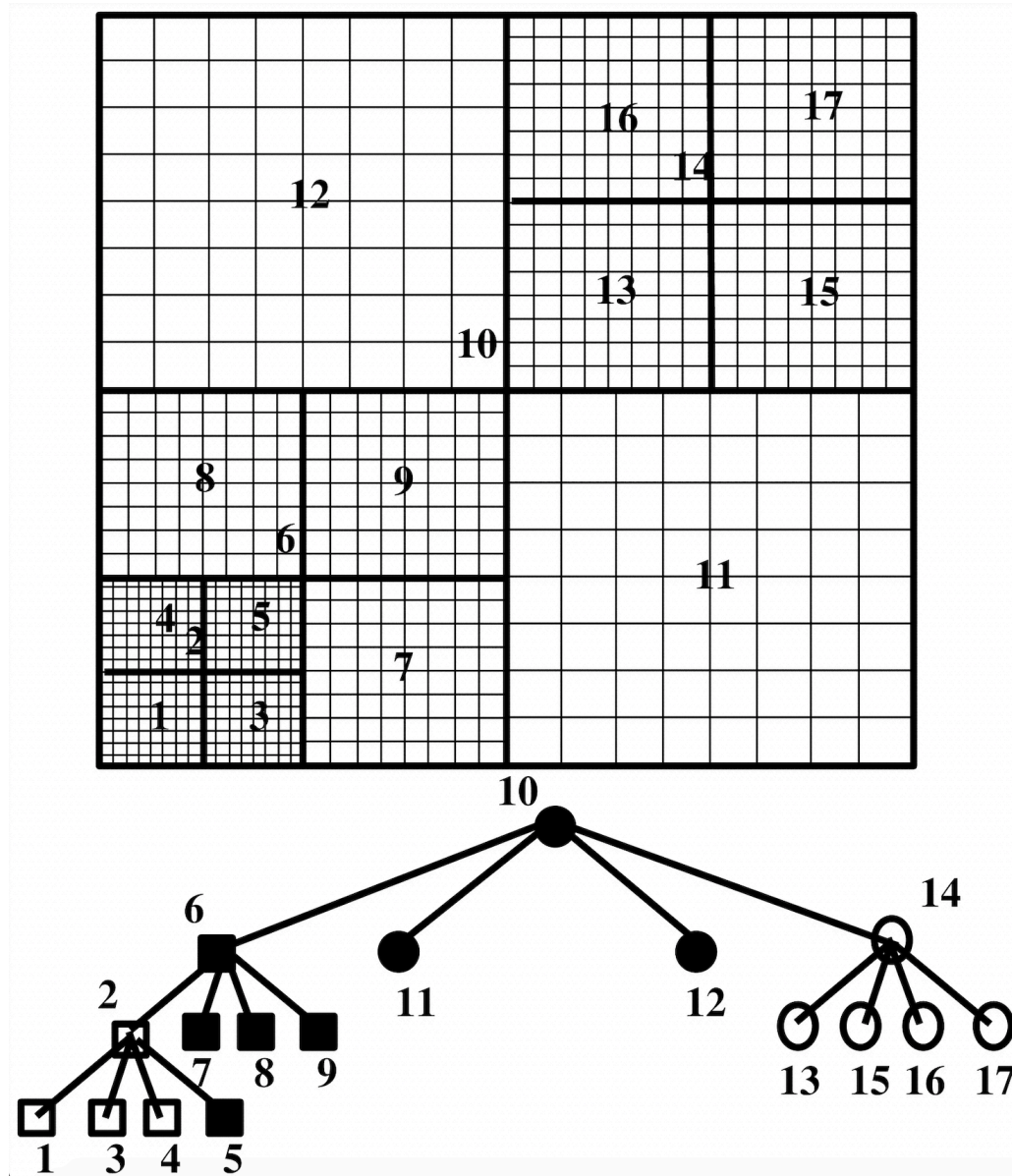


AMR in Action

- Eulerian Grid
- Adaptive Mesh Refinement
- Sink Particles
(Federrath, C. et al. 2010)



PARAMESH



Images: Fryxell et al. 2000

PARAMESH: MacNeice et al. 2000

What Does FLASH Do?

- Solves conservative hydrodynamical equations for each cell face.
- Uses EoS solver to derive remaining information.
- FLASH is modular. But an enormous number of calculations need to be completed each run.

$$N_{\text{blocks}} = 2^{3(n-1)}$$

n = refinement #

- Each block has 512 cells
- Each cell must be iterated through to solve hydrodynamical equations and gravity equation.

$$\begin{aligned} \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) &= 0 \\ \frac{\partial(\rho \mathbf{v})}{\partial t} + \nabla \cdot (\rho \mathbf{v} \mathbf{v}) &= \mathbf{g} - \frac{\nabla p}{\rho} \\ \frac{\partial(\rho E)}{\partial t} + \nabla \cdot [(\rho E + P)\mathbf{v}] &= \rho \mathbf{v} \cdot \mathbf{g} \end{aligned}$$

Is FLASH Appropriate to Use?

- Grid vs. Smooth-Particle Hydrodynamics (SPH)
(Agertz, O. 2007)
- Grid-codes can resolve shocks and flow instabilities
 - Important for SLR mixing
- Grids have issues with angular momentum conservation.
- Grids have issues with field discontinuities.

First Attempt: A Disk in Equilibrium

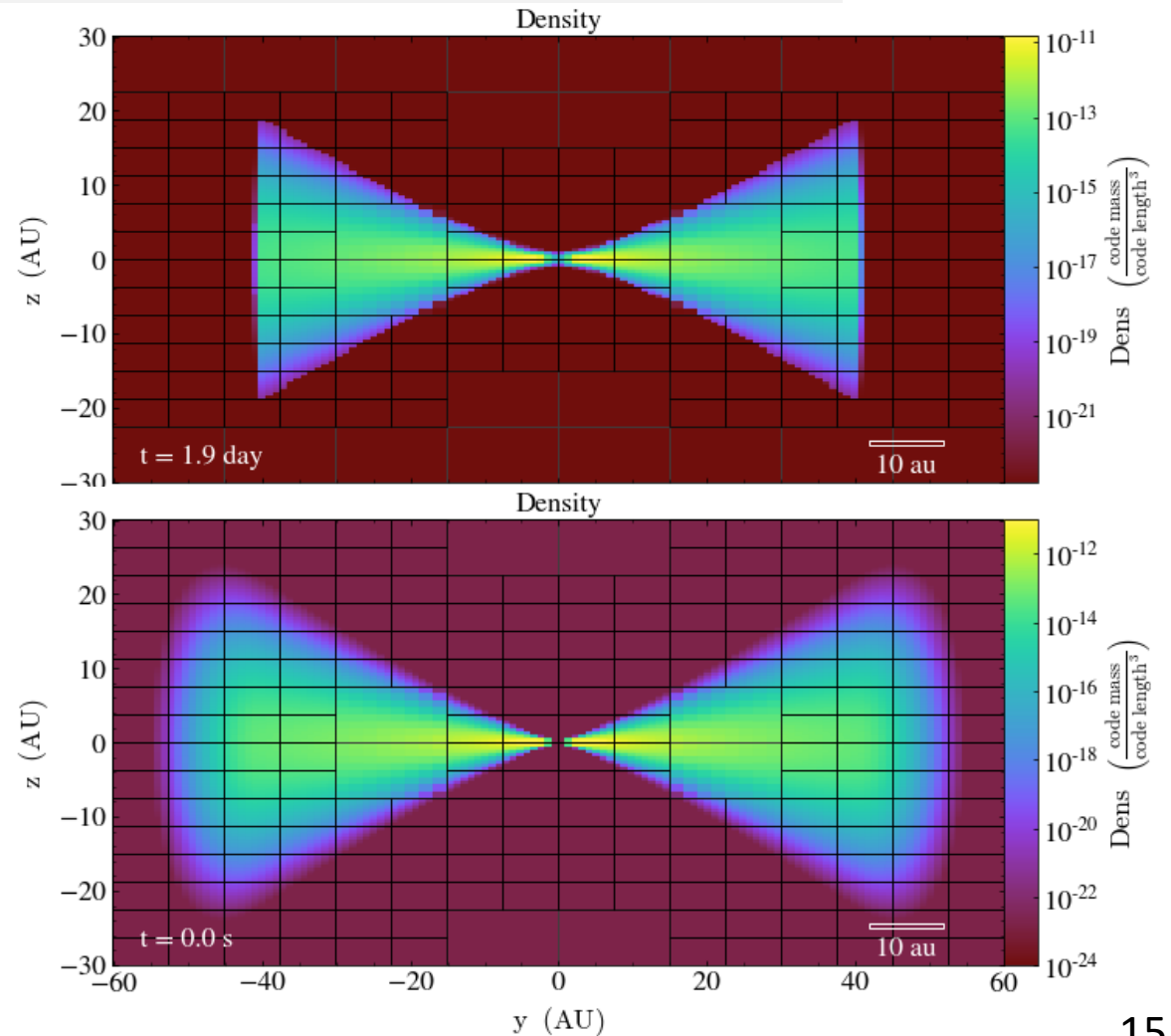
Initializing a Disk in Hydrostatic Equilibrium

- Distributed density according to Hayashi (1985)
 - Can change initial distribution easily in Simulation initialization file.
- Discontinuity at outer edge prevents disk evolution past a few time steps
- Add exponential decay to outer edge

$$\rho(r, z) = \rho_0 r^{-2.75} \exp\{-z^2 / z_0^2(r)\}$$

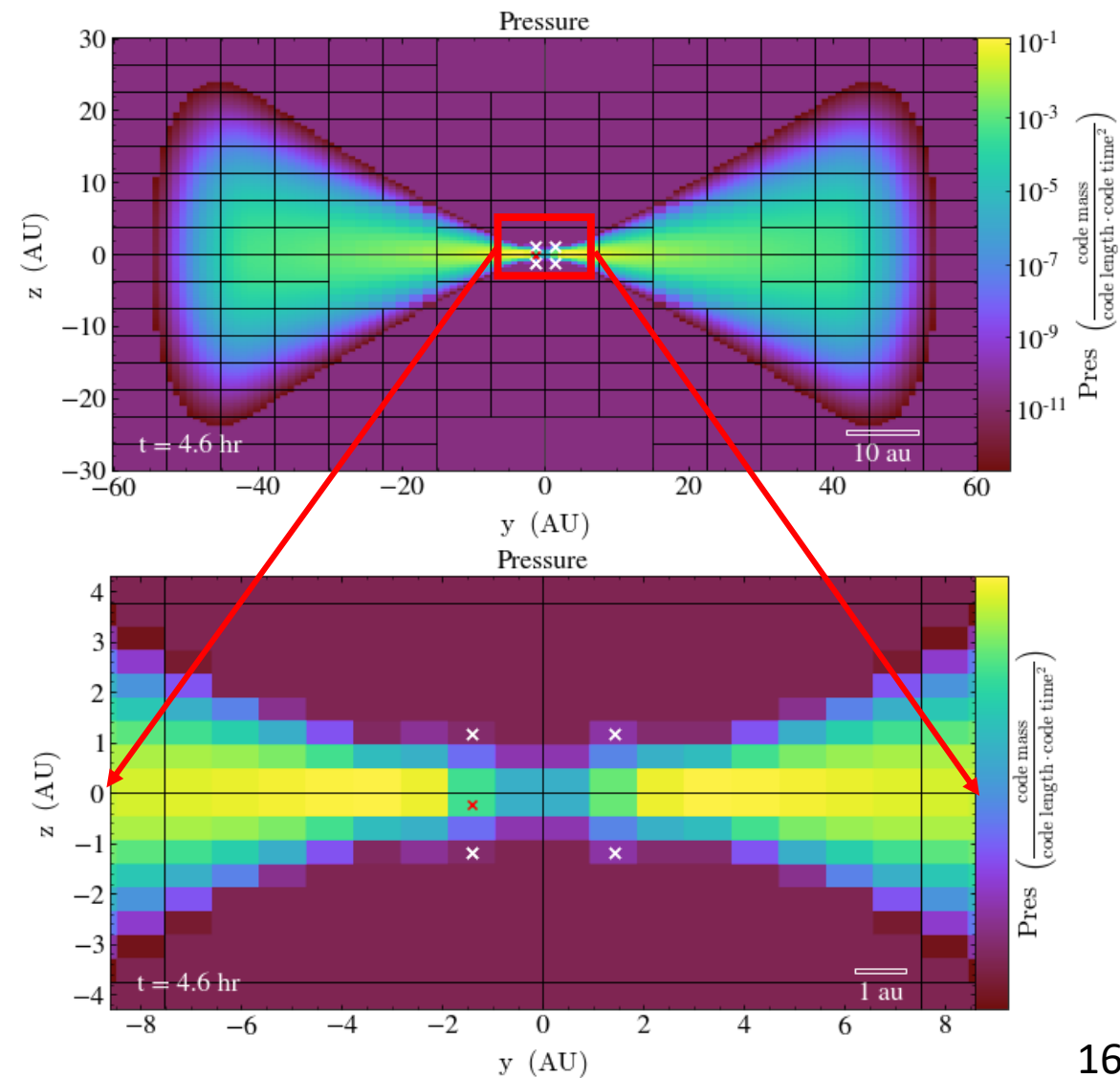
$$\rho_0 = 1.4 \times 10^{-9} \text{ g cm}^{-3}$$

$$z_0 = 0.0472 \times r^{5/4}$$



More Discontinuities!

- Even with buffer added to outer edge of disk: simulation is unable to progress more than a few time steps.
- Cells at inner boundary of disk become vacated, most likely the hydrodynamical solvers are failing at the discontinuity.

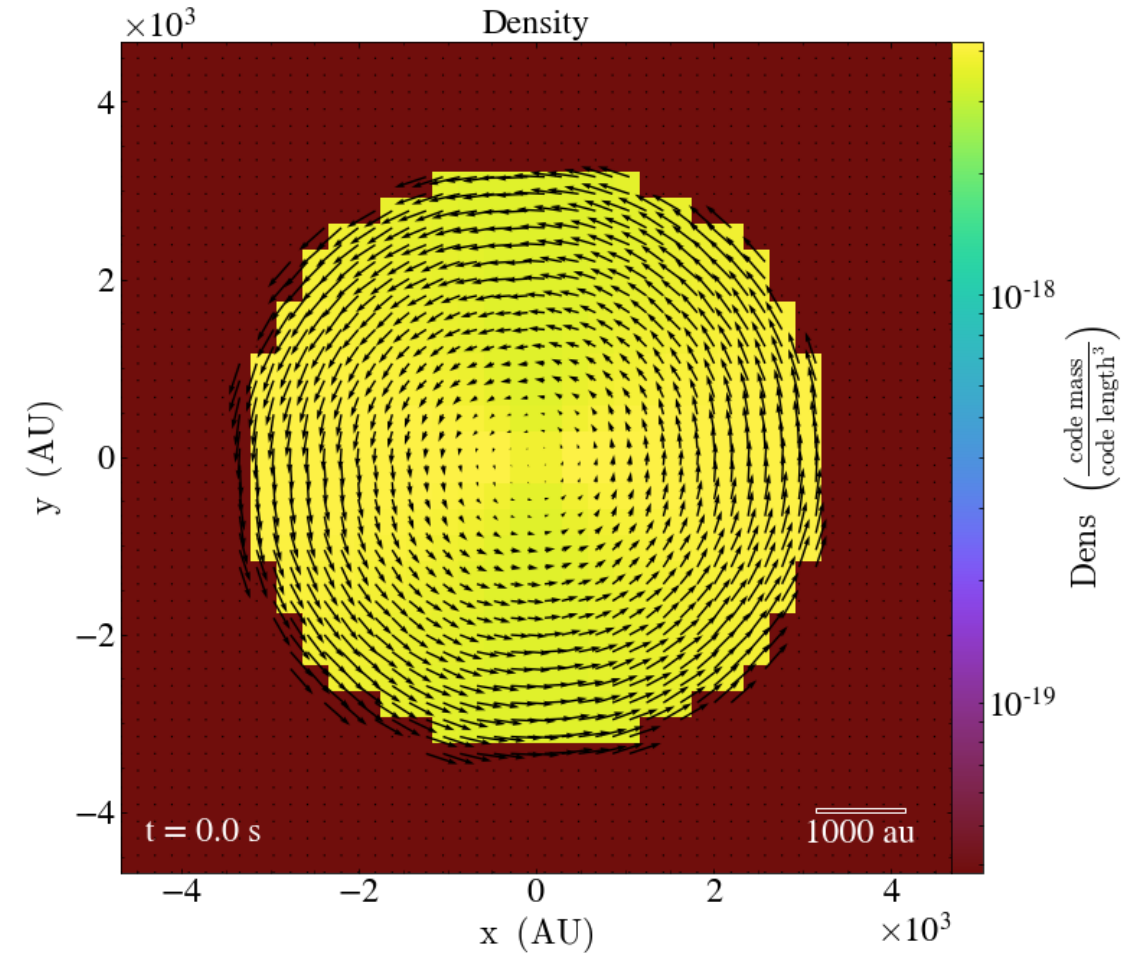


New Approach: A Disk From a Collapsing Nebula

A Collapsing Gas Sphere

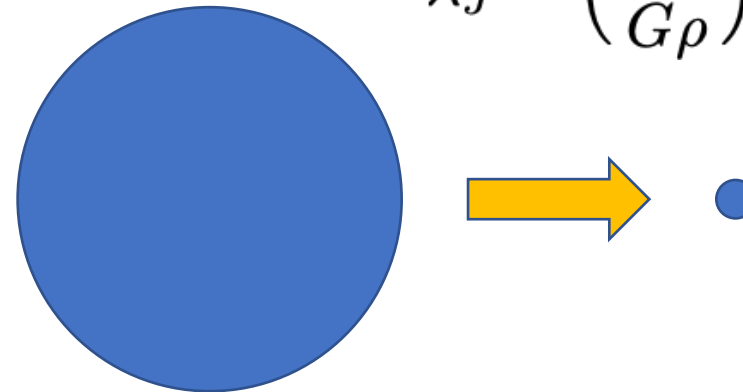
- Standard problem initializes 1 Solar Mass of self-gravitating, rotating, sphere of gas ~ 4000 AU across.
- Cloud collapses, forms a sink particle, relaxes into disk 2000 AU across

(Federrath, C. 2010; Boss & Bodenheimer 1979)



Scaling Down The Sphere

- In order to focus on a region closer to the desired few hundred AU as opposed to a few thousand, the standard problem can be scaled.
- Reduce all spatial parameters by a factor of 10.
 - x,y,z coordinates
 - **Jean's Length** (Jeans, J.H. 1902)
 - Density must increase by factor of 100
 - Total mass reduced by factor of 10



$$\lambda_J = \left(\frac{\pi c_s^2}{G \rho} \right)^{1/2}$$

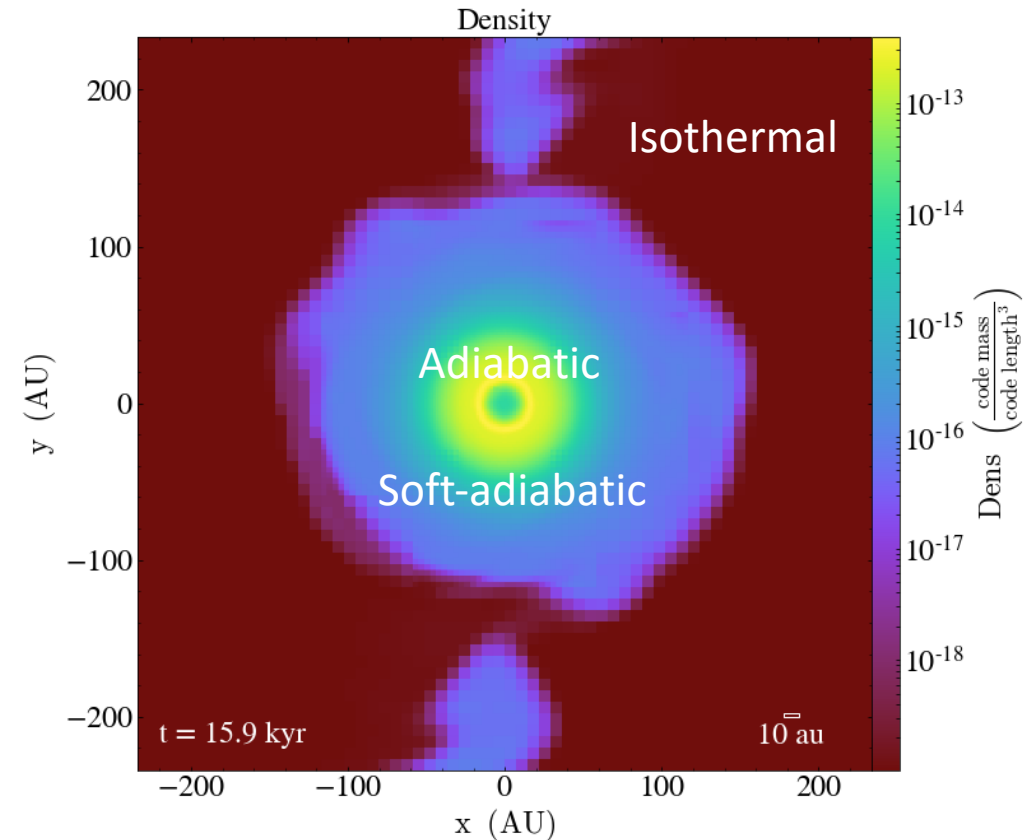
Understanding the Equation of State

- Determines compressibility of gas based on its density.

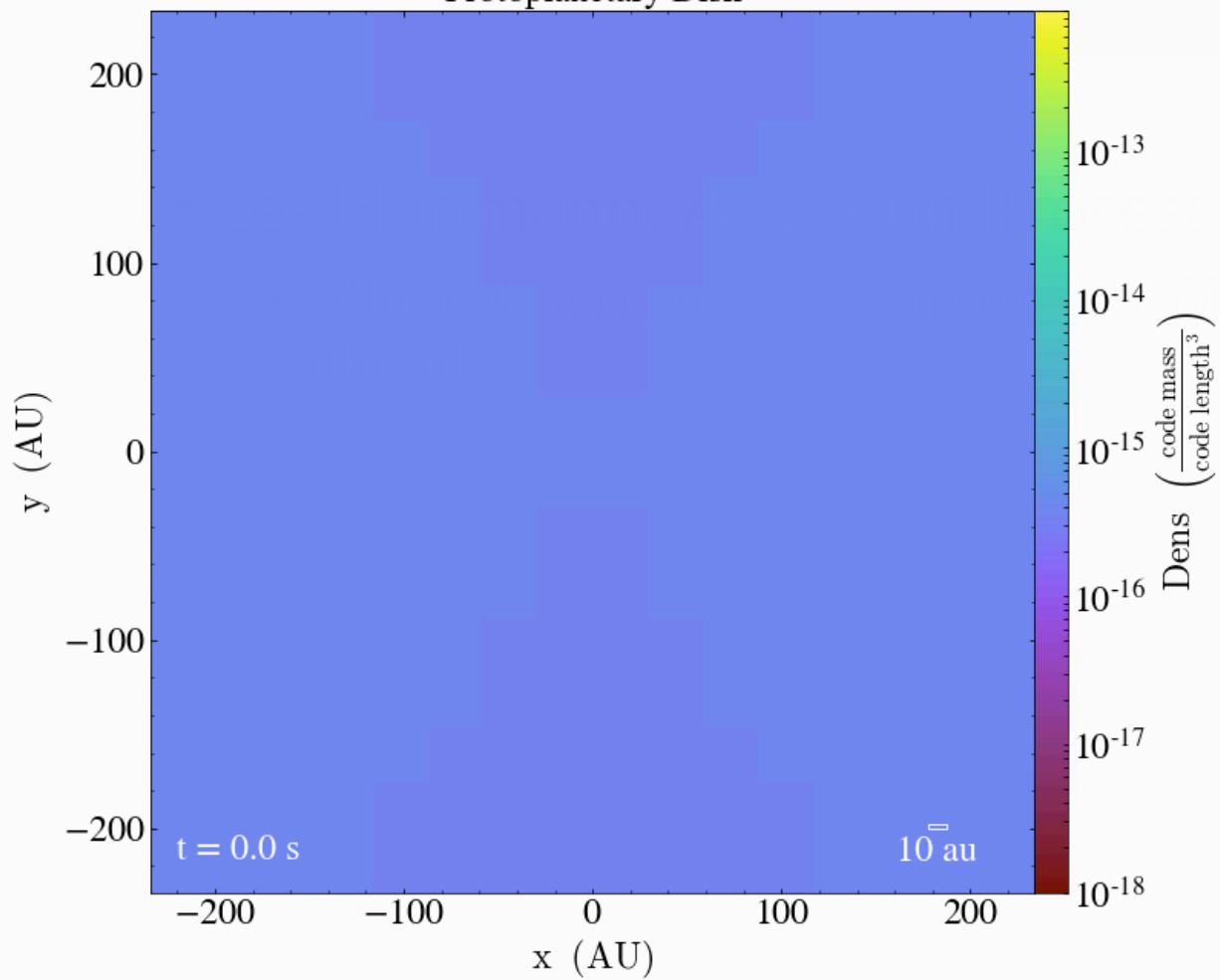
$$P = c_s^2 \rho^\Gamma$$

$$\Gamma = \begin{cases} 1 & \text{for } \rho \leq 2.5 \times 10^{-14} \text{ g cm}^{-3} \\ 1.1 & \text{for } \rho \leq 5.0 \times 10^{-13} \text{ g cm}^{-3} \\ 4/3 & \text{for } \rho > 5.0 \times 10^{-13} \text{ g cm}^{-3} \end{cases}$$

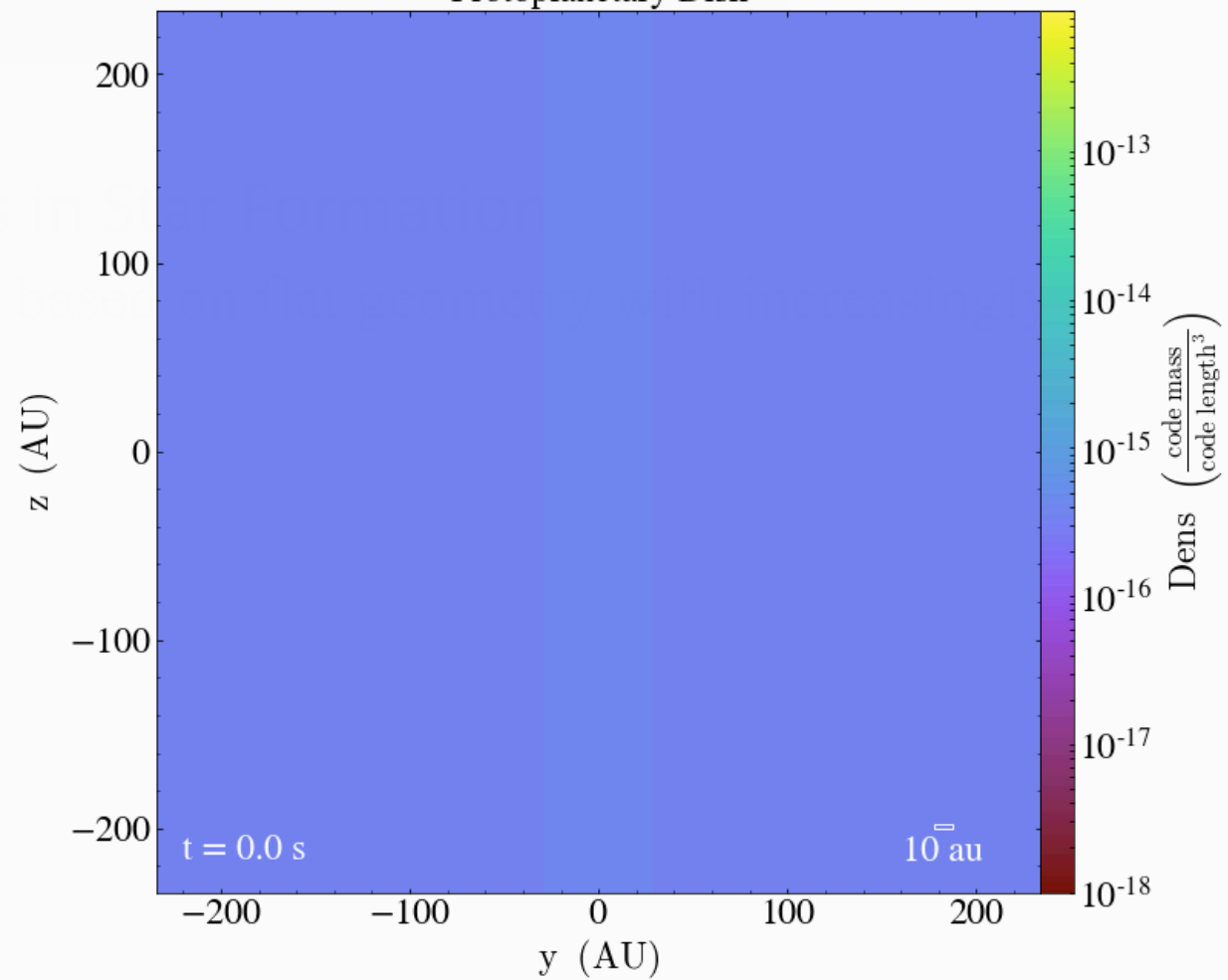
- Isothermal, adiabatic, all needs to be scaled too.



Protoplanetary Disk

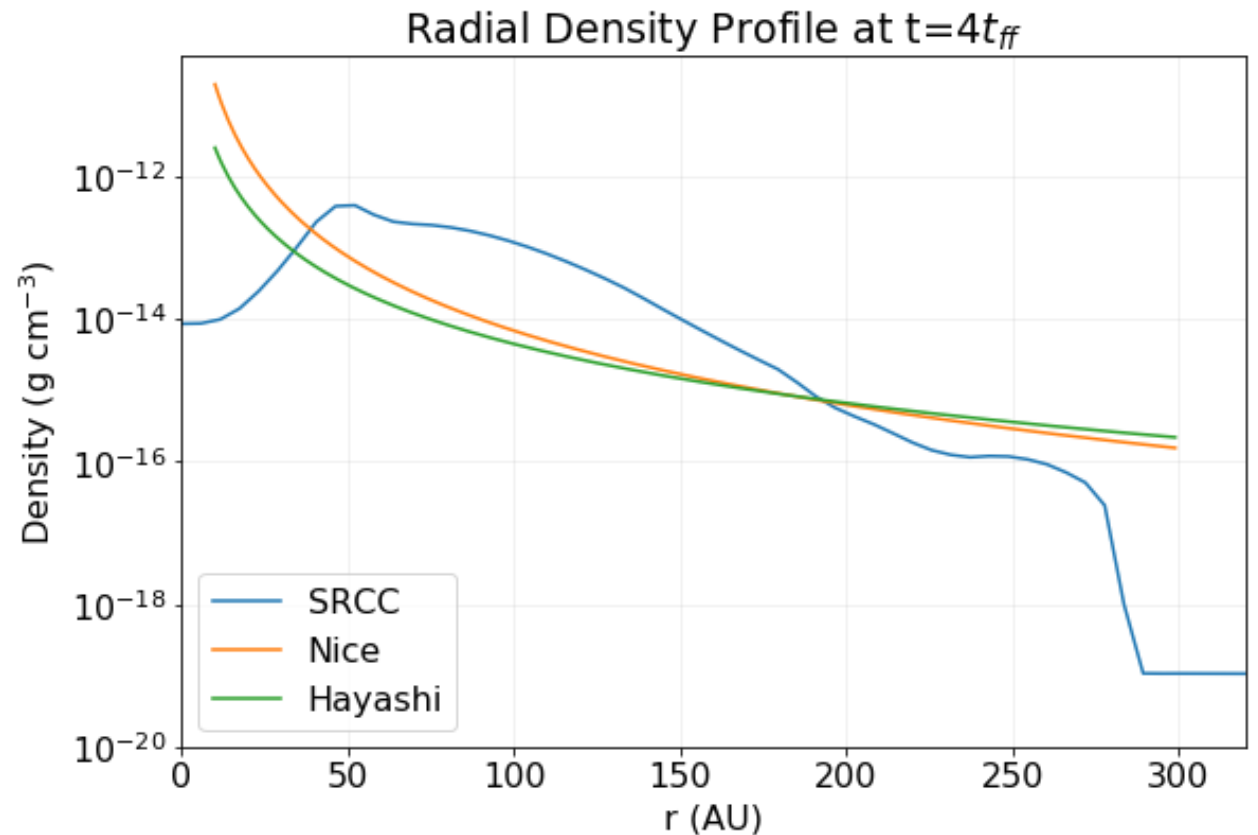


Protoplanetary Disk



How Density Compares to Models

- Diverges significantly from proposed disk models Hayashi and Nice
 - Models curtailed specifically for our Solar System
- Disk appears relaxed.



Pros & Cons of the Collapsing Sphere

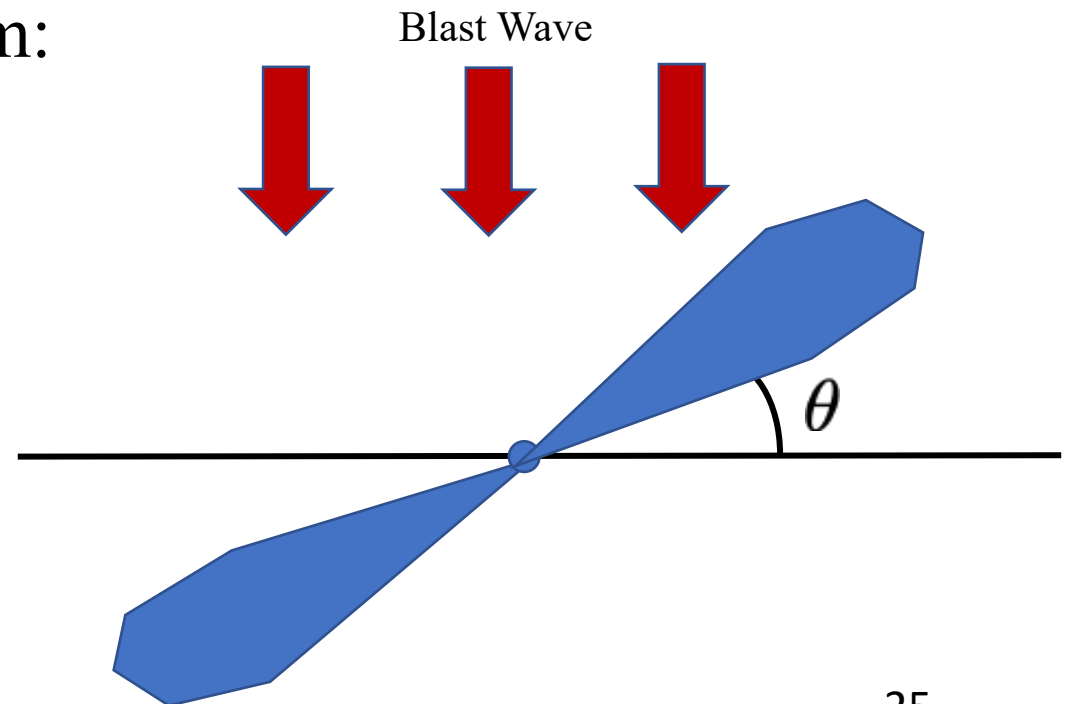
- Pros:
 - Easy to initialize
 - Less time for user to make minor adjustments
 - Easy to scale
- Cons:
 - Cannot manipulate disk structure directly.

Inclining the Disk

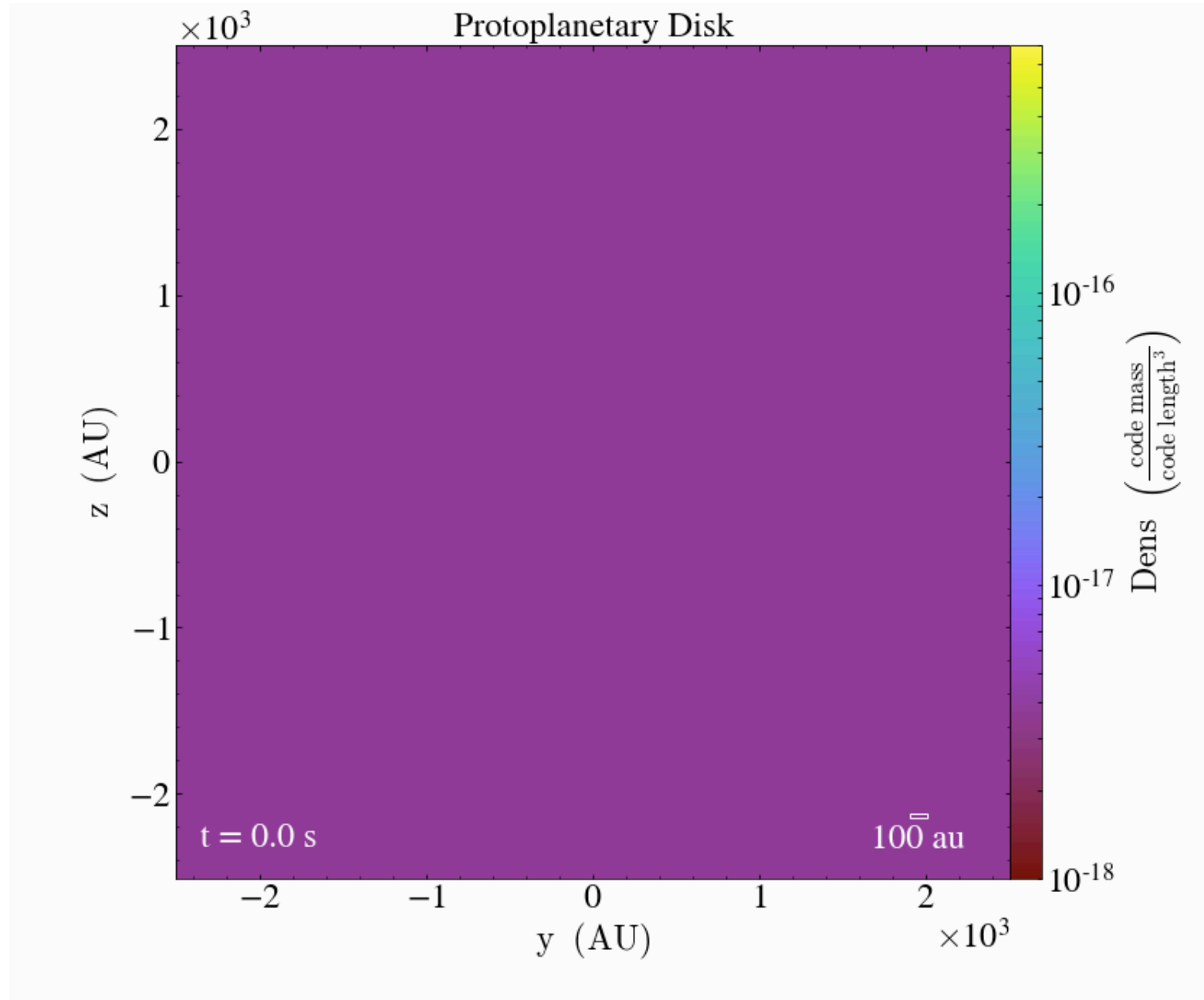
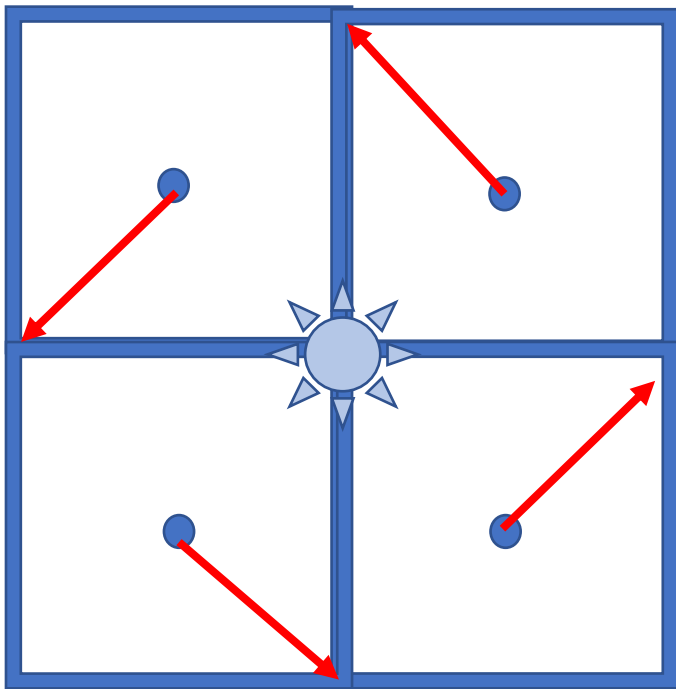
Inclining the Disk

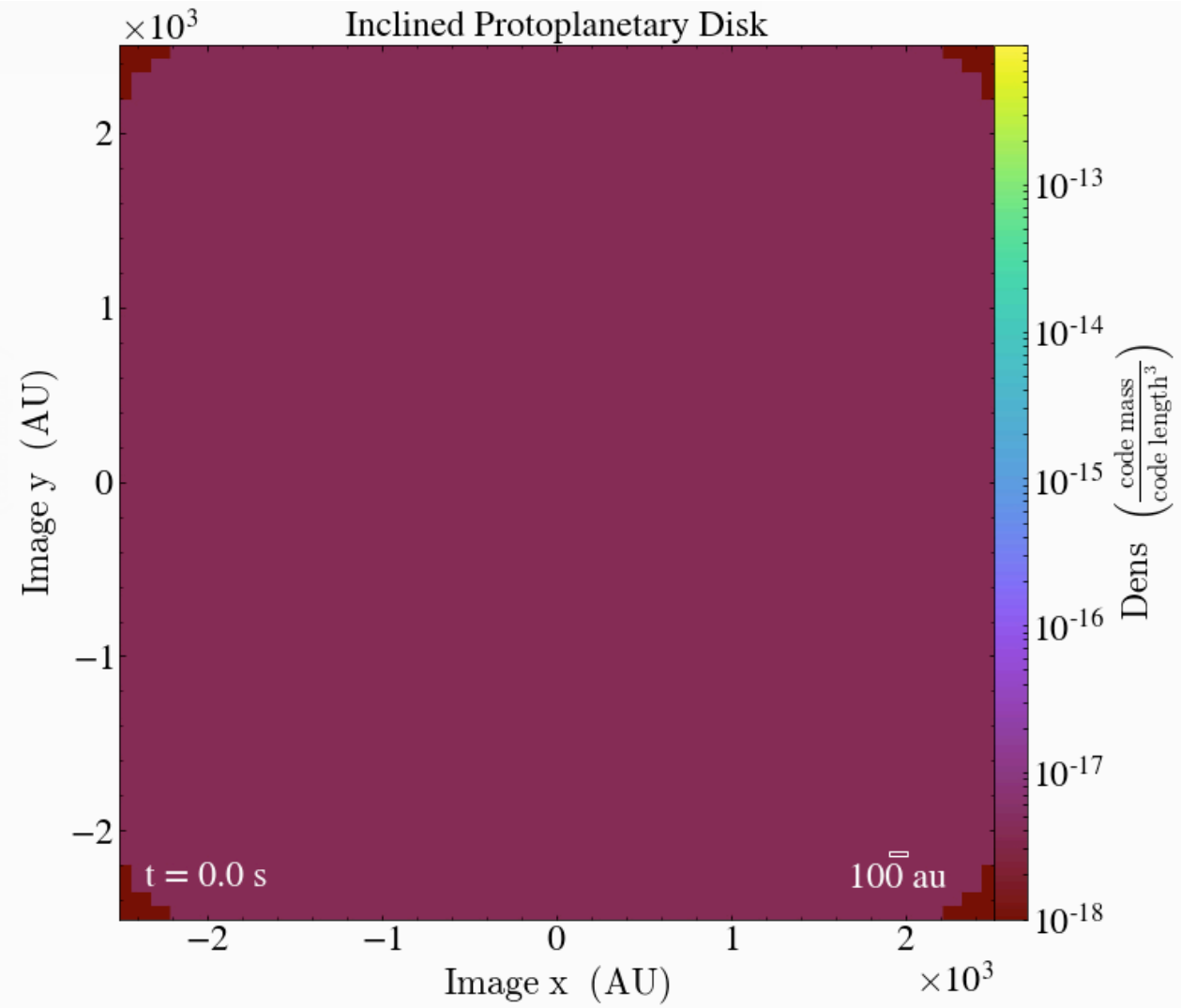
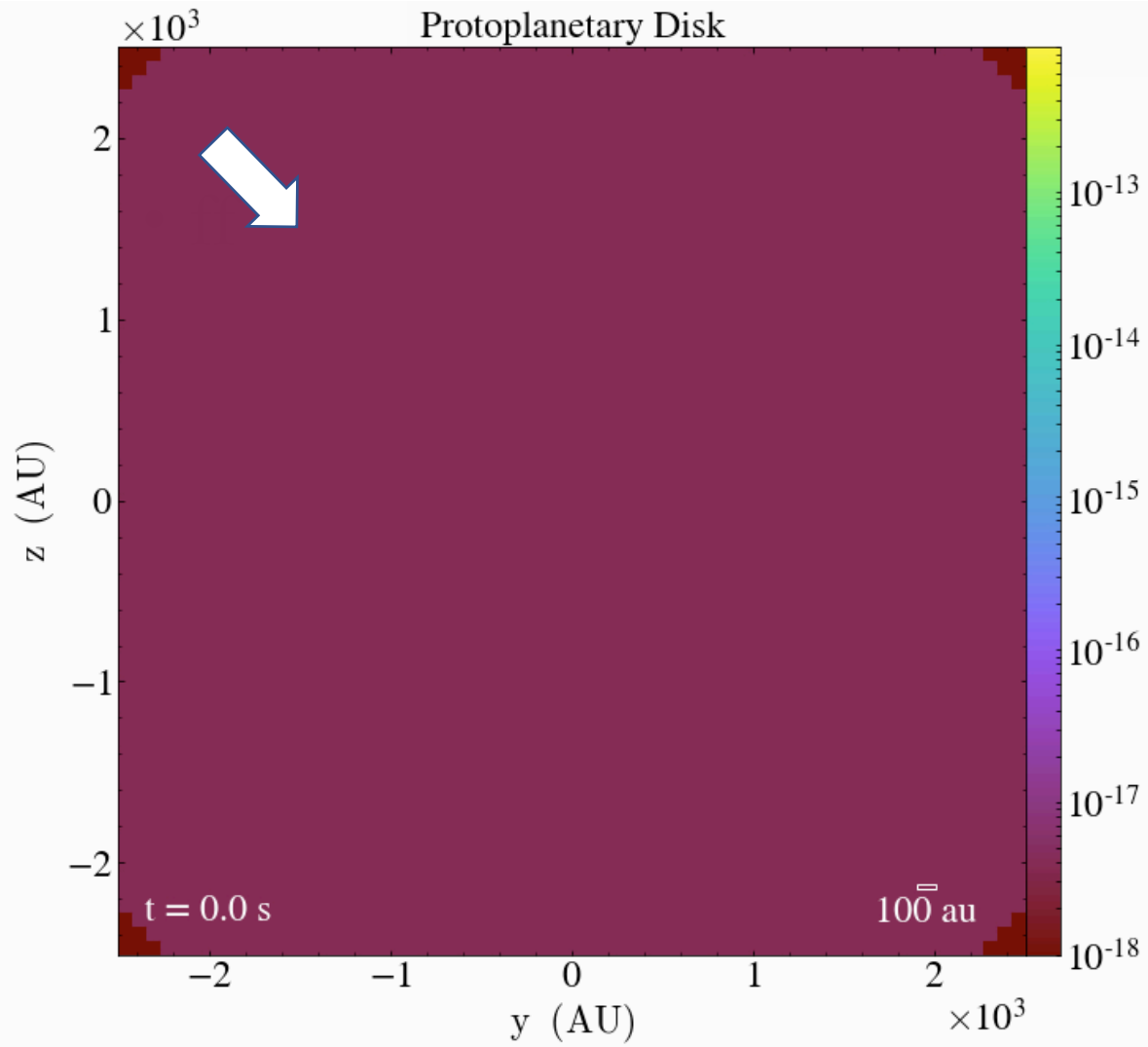
- Supernova can be introduced into the cubic simulation space through any one of the boundary faces.
- No easy way to introduce an inclined blast wave, so instead incline the disk by rotating initial coordinate system:

$$\begin{bmatrix} v'_x \\ v'_y \\ v'_z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & \sin(\theta) \\ 0 & -\sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}$$



Angular Momentum Error



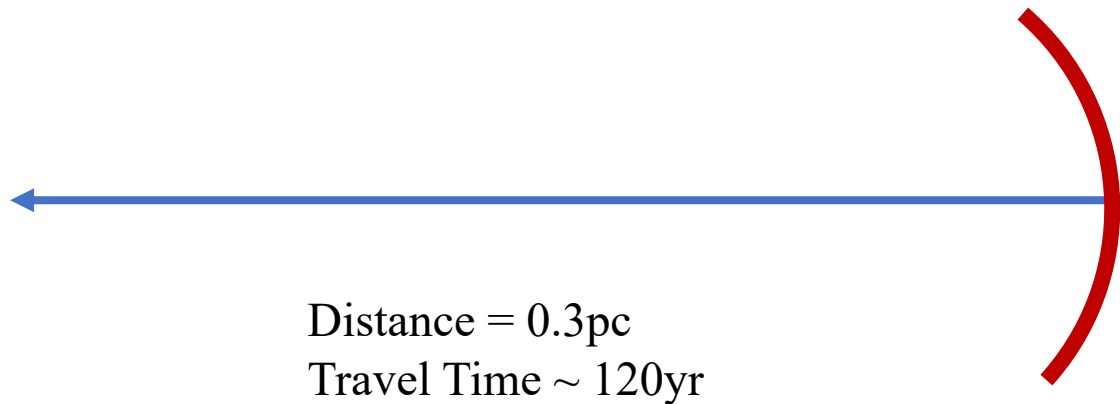


Modeling Supernova

- For how long is the disk in contact with the disk?

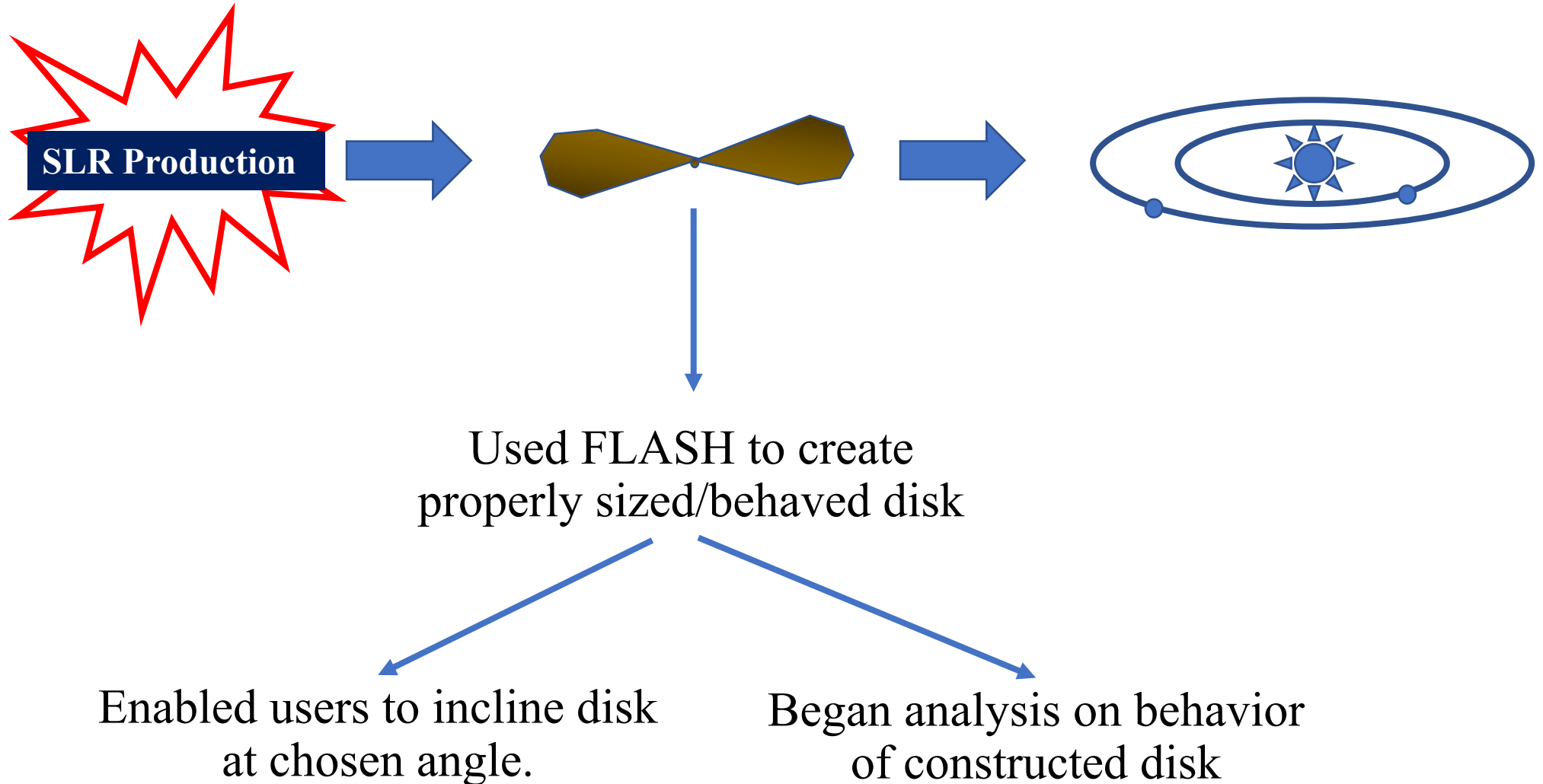
Matzner & McKee (1999), Ouellette et al. (2007)

$$\rho_{ej}(t) = \rho_* \left(\frac{t_*}{t_{trav}} \right)^3 \left(\frac{t_{trav}}{t + t_{trav}} \right)^3$$



Ejecta density will reduce by $1e-4$ in 2000 yr.

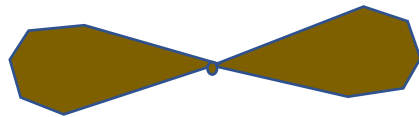
Results



Future Work



Modeling & introducing supernova blast wave
(Matzner & McKee 1999)



Angular momentum, gas accretion, etc.
More complex physics: radiative transfer, magnetic fields.



Simulating gas fragmentation and planet
formation using *Torch* (Wall, J. E. 2019)



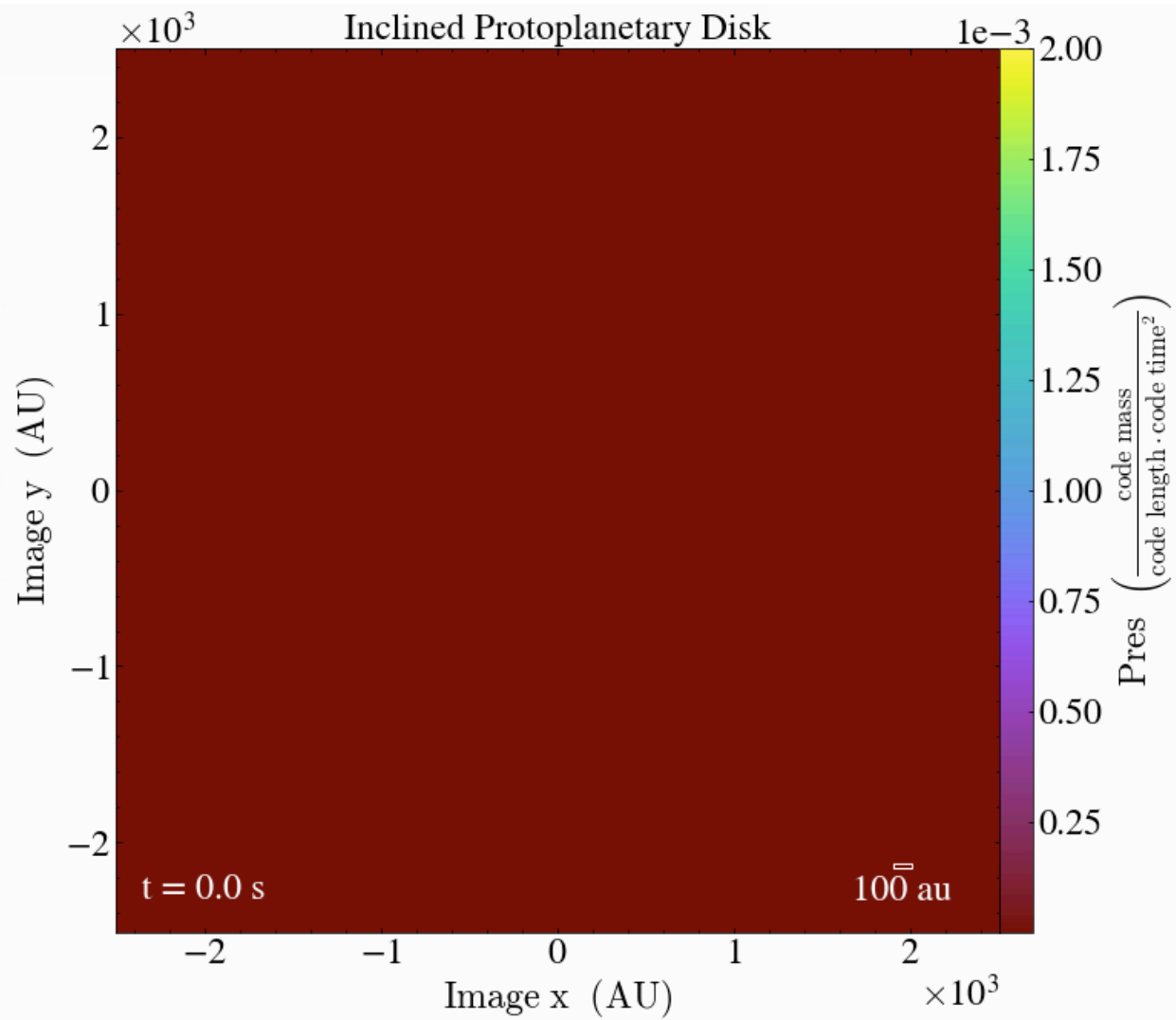
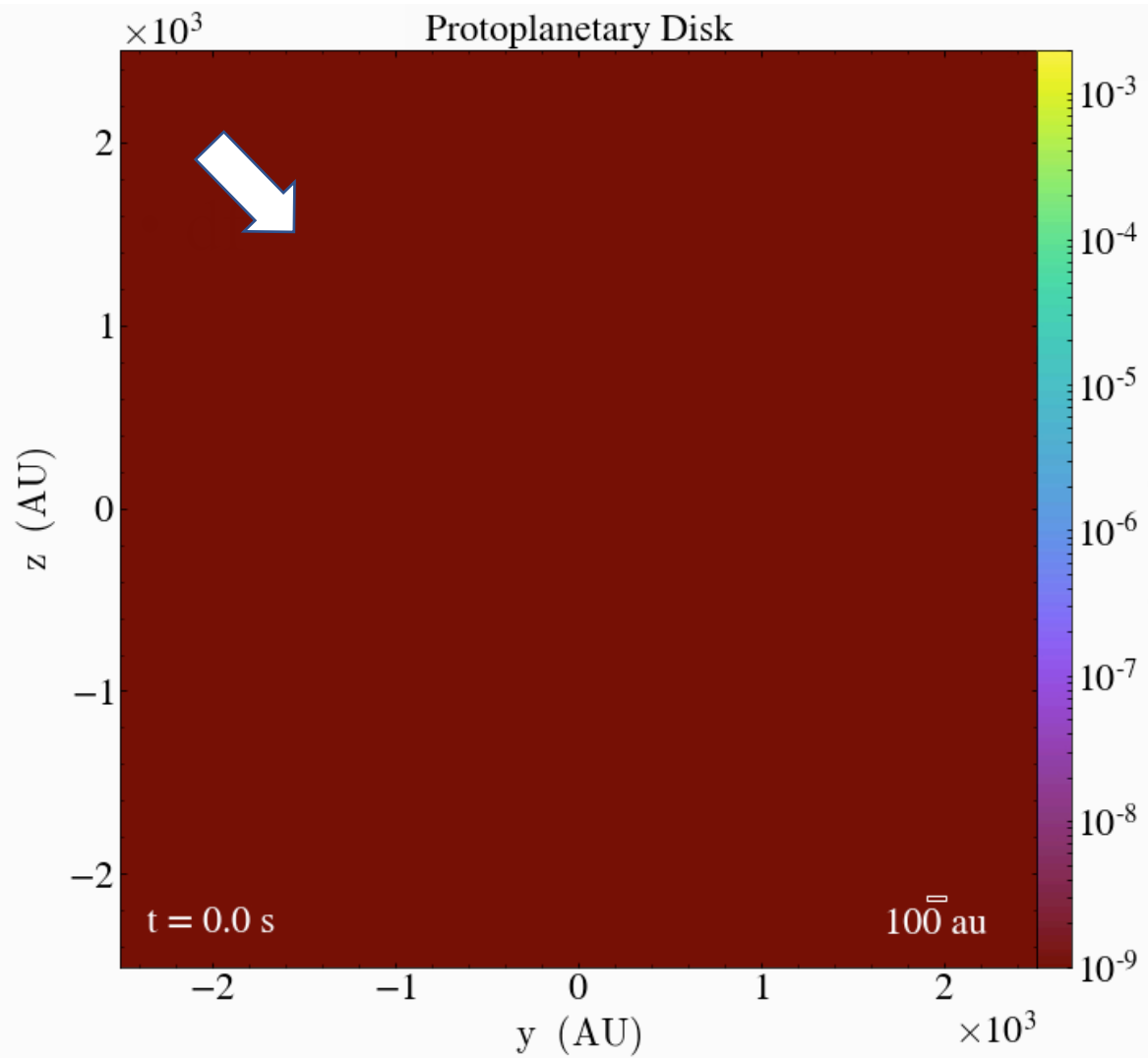
Thank You

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Results

- Successfully scaled down a FLASH test problem to create a ~ 0.09 solar mass sink-particle surrounded by a ~ 0.01 gaseous disk.
- Enabled users to incline the rotating cloud's velocity vectors and any defined density inhomogeneities (and therefore inclining the resulting disk).
- Additional questions: quantifying angular momentum loss and mass accretion dependency on inclination, density profile evolution, etc.
- Ultimately created a framework on which the introduction of a supernova blast can be tested, along with more complex physics by implementing FLASH's physics modules.

Is FLASH the best?

Block Requirement Calculation

$$N_blocks =$$

$$2^{3(n-1)}$$

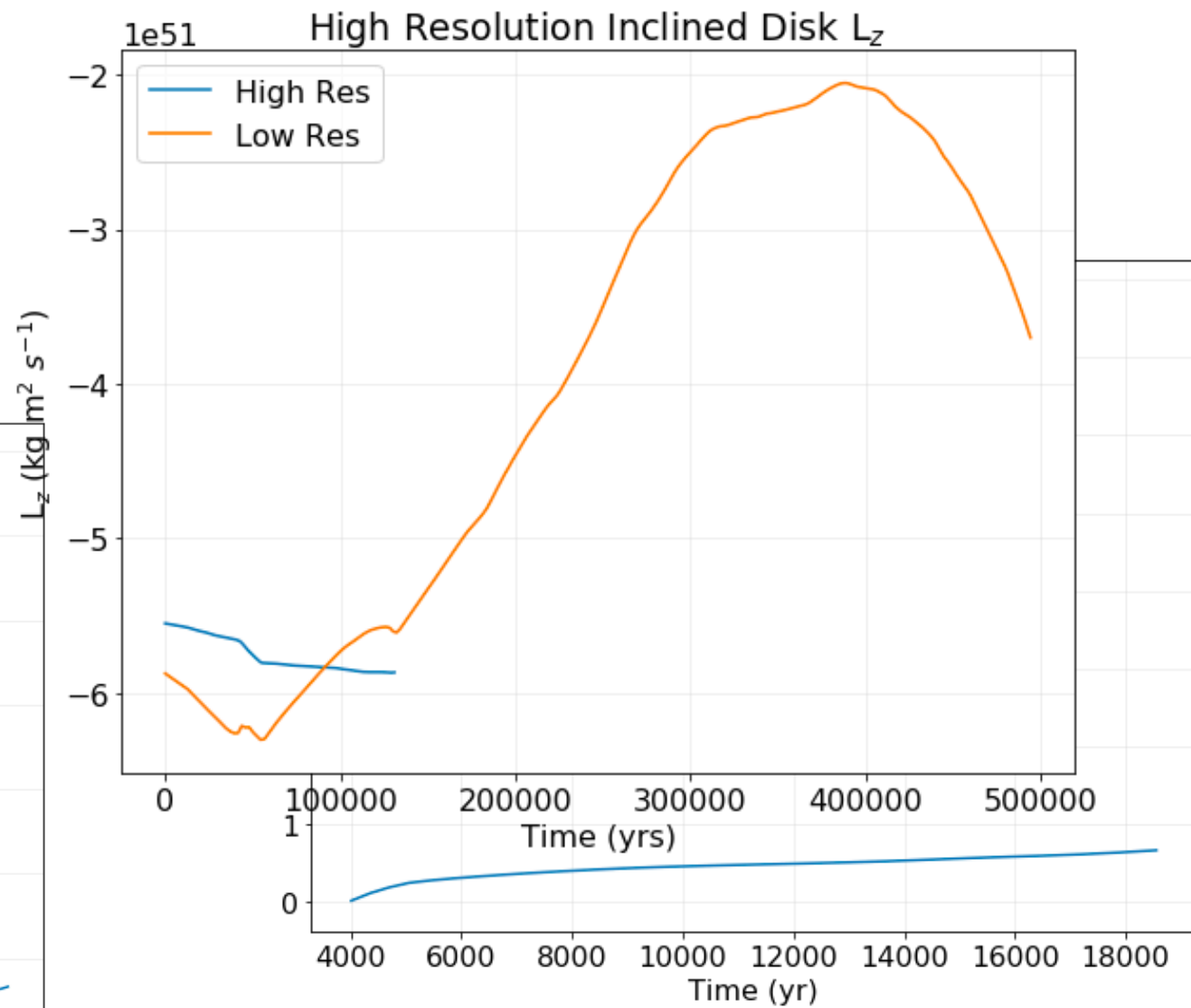
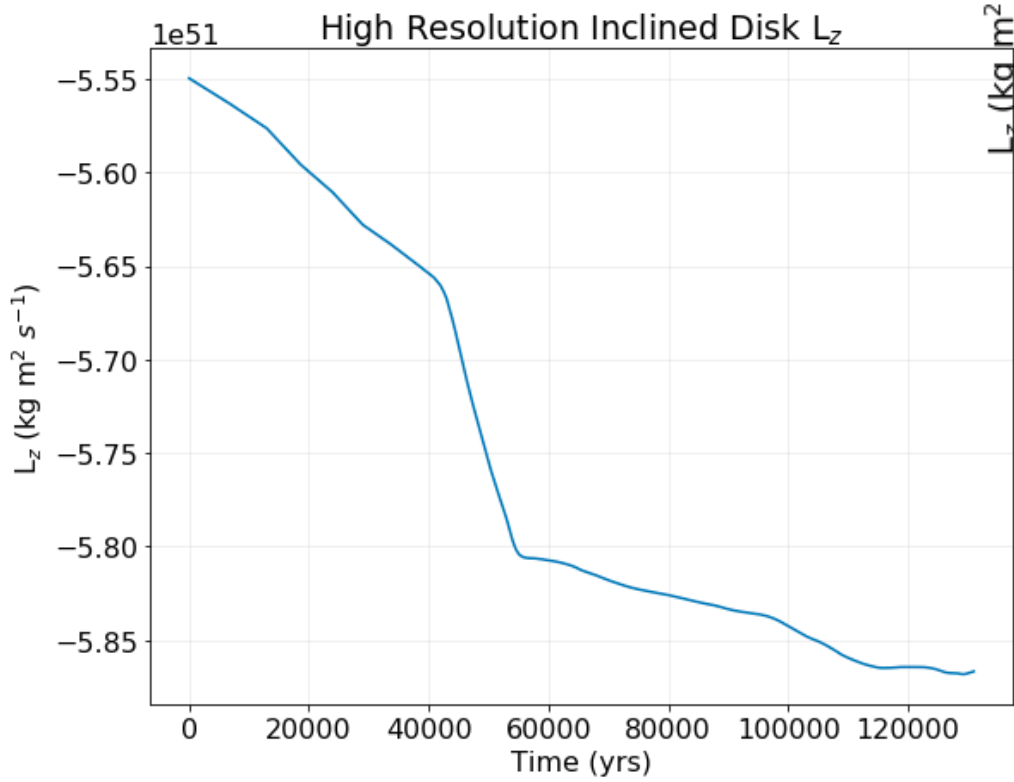
With n = refinement number

$$N_cells = 2^{3(n-1)} * 8^3$$

$$N_flow\ solves = 2^{3(n-1)} * 8^3 * 6$$

At ~200 blocks per processor, the lowest resolution runs can be completed on a modestly robust laptop

I used Draco computer cluster, build OpenMPI by hand (Necessary for multi-node computing)



Banerjee & Pudritz (2006)

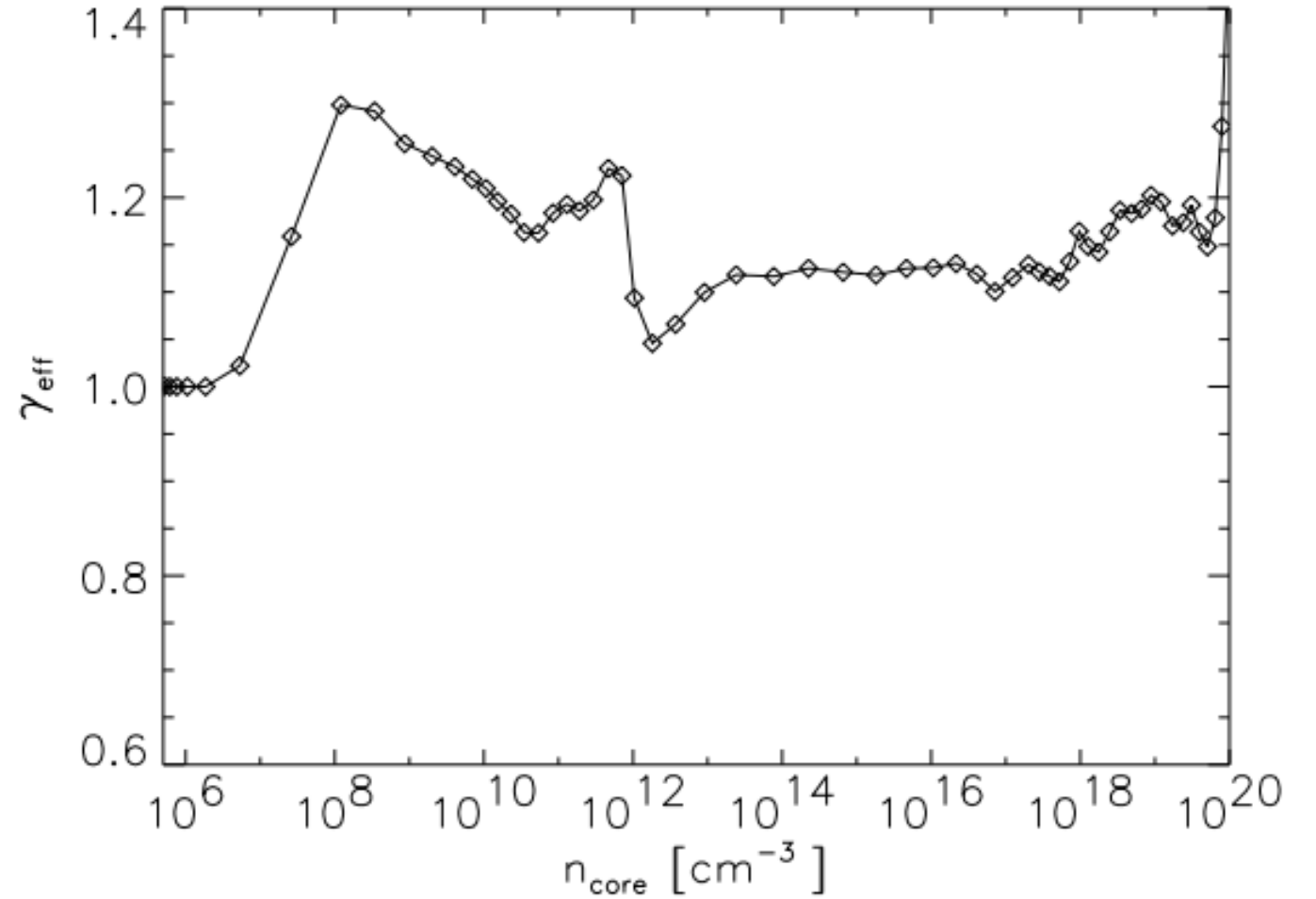


Fig. 9.— The effective equation of state, $\gamma_{\text{eff}} = d \ln p / d \ln \rho$ as a function of the core density n_{core} . Cooling by molecular line emission is effective until $n \sim 10^{7.5} \text{ cm}^{-3}$ above which the effective EOS reflects the molecular cooling ability in the different density/temperature regimes as well as the appearance of shocks.

$$v_* = \sqrt{\frac{E_{in}}{M_{ej}}} = 2240 \left(\frac{E_{in}}{10^{51} \text{erg} \cdot s} \right)^{0.5} \times \left(\frac{M_{ej}}{10 M_{\odot}} \right)^{-0.5} \text{km} \cdot \text{s}^{-1} \quad (8)$$

Where E_{in} is the initial energy of the supernova and M_{ej} is the total mass of the ejected material. The resulting velocity of the material is taken to be the radial velocity of the outwardly expanding sphere and is constant throughout the journey of the material from its star to the target. The elapsed time for material to escape the star:

$$t_* = R_* \sqrt{\frac{M_{ej}}{E_{in}}} = 1.552 \times 10^4 \left(\frac{R_*}{50 R_{\odot}} \right) \times \left(\frac{E_{in}}{10^{51} \text{erg} \cdot s} \right)^{-0.5} \times \left(\frac{M_{ej}}{10 M_{\odot}} \right)^{0.5} \text{s} \quad (9)$$

The initial density and pressure of of the ejecta:

$$\rho_* = \frac{M_{ej}}{R_*^3} = 4.72 \times 10^{-4} \left(\frac{M_{ej}}{10 M_{\odot}} \right) \times \left(\frac{R_*}{50 R_{\odot}} \right)^{-3} \text{g} \cdot \text{cm}^{-3} \quad (10)$$

$$P_* = \frac{E_{ej}}{R_*^3} = 2.278 \times 10^{13} \left(\frac{E_{in}}{10^{51} \text{erg} \cdot s} \right)^{0.75} \times \left(\frac{R_*}{50 R_{\odot}} \right)^{0.75} \times \left(\frac{M_{ej}}{10 M_{\odot}} \right)^{-1} \text{ergs} \cdot \text{cm}^3 \quad (11)$$

Where R_* is the radius of the exploding star.

The ratio of the escape time t_* to the travel time between the explosion and target, $t_{trav} = d/v_*$, scales the blast wave's density and pressure. In addition, as the blast wave passes over the disk at $t = 0$, the density, pressure, and wave velocity decrease as time elapsed increases:

$$\rho_{ej}(t) = \rho_* \left(\frac{t_*}{t_{trav}} \right)^3 \left(\frac{t_{trav}}{t + t_{trav}} \right)^3 \quad (12)$$

$$P_{ej} = P_* \left(\frac{t_*}{t_{trav}} \right)^4 \left(\frac{t_{trav}}{t + t_{trav}} \right)^4 \quad (13)$$

$$v(t) = v_* \left(\frac{t_{trav}}{t + t_{trav}} \right) \quad (14)$$

As of yet, the supernova blast has not been simulated here. By defining the +z boundary as an inflow boundary, I will be able to control the velocity, pressure, and density behavior of gas as it begins to move into the simulation-space.

- Oscar Agertz
<http://adsak>

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \nabla \cdot \mathbf{v} = \mathbf{g} - \frac{\nabla p}{\rho}$$

$$\frac{\partial e}{\partial t} + \mathbf{v} \cdot \nabla e + \frac{p}{\rho} \nabla \mathbf{v} = 0$$

$$E = \epsilon + \frac{1}{2} |\mathbf{v}|^2$$

$$\frac{P}{\rho} = c_s^2 = \frac{k_b T}{m}$$

$$E = \epsilon + \frac{1}{2} |\mathbf{v}|^2$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \quad \frac{\partial (\rho \mathbf{v})}{\partial t} + \nabla \cdot (\rho \mathbf{v} \mathbf{v}) = \mathbf{g} - \frac{\nabla p}{\rho} \quad \frac{\partial (\rho E)}{\partial t} + \nabla \cdot [(\rho E + P) \mathbf{v}] = \rho \mathbf{v} \cdot \mathbf{g}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \mathbf{v} = 0 \quad \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} + \frac{\nabla p}{\rho} = \mathbf{g} \quad \frac{\partial e}{\partial t} + \mathbf{v} \cdot \nabla e + \frac{p}{\rho} \nabla \cdot \mathbf{v} = 0$$