Zero-Point Energy and 1D Lattice Structures

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In the early part of the 20th century, Max Planck, Albert Einstein, Otto Stern, and other leading-edge physicists hypothesized and developed the foundation of zero-point energy and quantization as a mathematical construct for describing forces in electromagnetism. The development of quantum mechanics, namely the Heisenberg uncertainty principle and quantum electrodynamics, reintroduced quantization and zero-point energy as a basis for the theory; quantum mechanics essentially predicts that all of space is filled with electromagnetic zero-point fluctuations. Physical evidence for zero-point energy has been found in the quantum electrodynamic phenomenon of the Casimir effect predicted by Hendrik Casimir in 1948, and over the past century many more developments, questions, and hypotheses have manifest due to this mysterious force.

I. INTRODUCTION

Zero-point energy (ZPE) can be thought of as a limitless source of potential energy that exists throughout the universe; in essence, ZPE describes the ground state of a quantum mechanical system that exhibits a fluctuating finite energy due to Heisenberg's uncertainty principle. Quantum mechanics predicts that all of space must be filled with electromagnetic zero-point fluctuations (also called the zero-point field) creating a universal sea of ZPE [1].

The Heisenberg uncertainty principle states that for a particle like an electron, the more precisely one measures its position, the less exact one can measure its momentum and vice versa. This uncertainty reflects an intrinsic quantum energy fluctuation in the wave nature of quantum systems and leads to ZPE, which essentially is the energy that remains when all other energy is removed from a system, i.e. a vacuum.

Max Planck, Albert Einstein, and Otto Stern worked in the early part of the 20th century to show this mathematically, and physical evidence for ZPE has been found in the quantum electrodynamic phenomenon of the Casimir effect predicted by Hendrik Casimir in 1948 [1].

While the original concept of ZPE originated with Planck, Nernst, Einstein, and others, the original foundation for ZPE has been reworked and rethought over the past century by physicists like Georges Lemaître and Edward Tryon, and continues to be investigated and re-understood today.

II. QUANTUM HARMONIC OSCILLATOR

The quantum harmonic oscillator is one of the foundational problems of quantum mechanics in understanding complex modes of atomic vibration in molecules and solid lattices. Using a quantum harmonic oscillator to describe ZPE is useful in that classically, a harmonic oscillator can always be brought to rest, whereas a quantum harmonic oscillator always has a ZPE due to Heisenberg's principle

$$\begin{aligned} \Delta x \Delta p &\geq \tfrac{\hbar}{2} \\ \Delta E \Delta t &\geq \tfrac{\hbar}{2} \end{aligned} \tag{1}$$

Classically, the potential for a harmonic oscillator can be written in terms of Schrödinger's equation

$$\begin{aligned} V(x) &= \tfrac{1}{2}kx^2 \\ -\tfrac{\hbar^2}{2m}\tfrac{d^2\Psi}{dx^2} + (\tfrac{1}{2}kx^2)\Psi &= E\Psi \end{aligned} \tag{2}$$

where k is the spring constant. By using raising and lowering operators, Schrödinger's equation can be written

$$\hat{H}\Psi = \hbar\omega(\hat{a}_-\hat{a}_+ - \tfrac{1}{2})\Psi = E\Psi \tag{3}$$

To find the mean energy for any mode, one can use the partition function and geometric sums to show

$$\langle H \rangle = \langle E \rangle = (\tfrac{1}{e^{\beta\hbar\omega}-1} + \tfrac{1}{2})\hbar\omega = (\langle n \rangle + \tfrac{1}{2})\hbar\omega \tag{4}$$

where $\langle n \rangle$ is the average occupancy number, β is Boltzmann's constant, and $\frac{\hbar\omega}{2}$ is the zero-point energy.

By taking the original Hamiltonian and lowering the energy to the ground state, i.e. $\hat{a}_-\Psi_0 = 0$ where Ψ_0 is the ground state wavefunction for the quantum harmonic oscillator, it can be written

$$\Psi_0 = (\tfrac{m\omega}{\pi\hbar})^{\frac{1}{4}} e^{-\frac{m\omega x^2}{2\hbar}} \tag{5}$$

Plugging this result back into the Schrödinger equation gives us the ground state zero-point energy of $\frac{\hbar\omega}{2}$. This is the smallest energy level allowed by the uncertainty principle and it demonstrates how physical systems like molecules in a lattice cannot have zero energy even at absolute zero temperatures.

The general solution to the Schrödinger equation leads to a sequence of evenly spaced energy levels in a potential characterized by a quantum number n using

$$E_n = (n + \tfrac{1}{2})\hbar\omega \tag{6}$$

where ω can be written $\sqrt{\frac{k}{m}}$. The equally spaced set of allowed vibrational energy levels observed for a quantum harmonic oscillator is not expected classically, where all energies would be possible. The quantization of the energy levels of the harmonic oscillator is a constant value, $\hbar\omega$, and at $n = 0$ there exists a zero-point energy of $\frac{\hbar\omega}{2}$. All vibrational energies, down to zero, are possible in the classical oscillator case, and so feature is also unique to quantum oscillators.

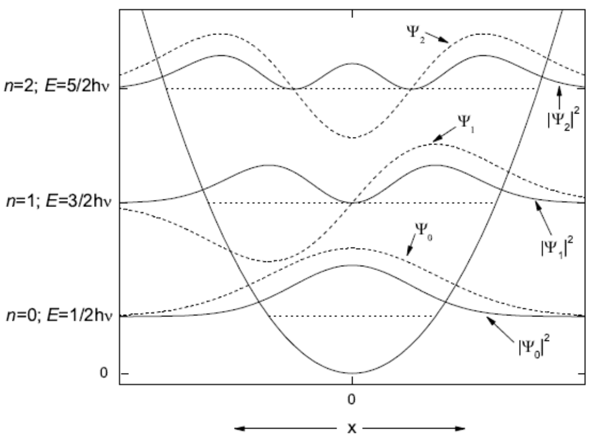

Figure 1: First three energy levels of quantum harmonic oscillator with potential and wave functions [2].

One can also see that, in general, the quantum harmonic oscillator will have $n + 1$ peaks, with n minima which correspond to nodes of zero probability. At large energies, the distance between the peaks become smaller than the Heisenberg uncertainty principle allows for observation and the quantum oscillator approaches a classical approximation.

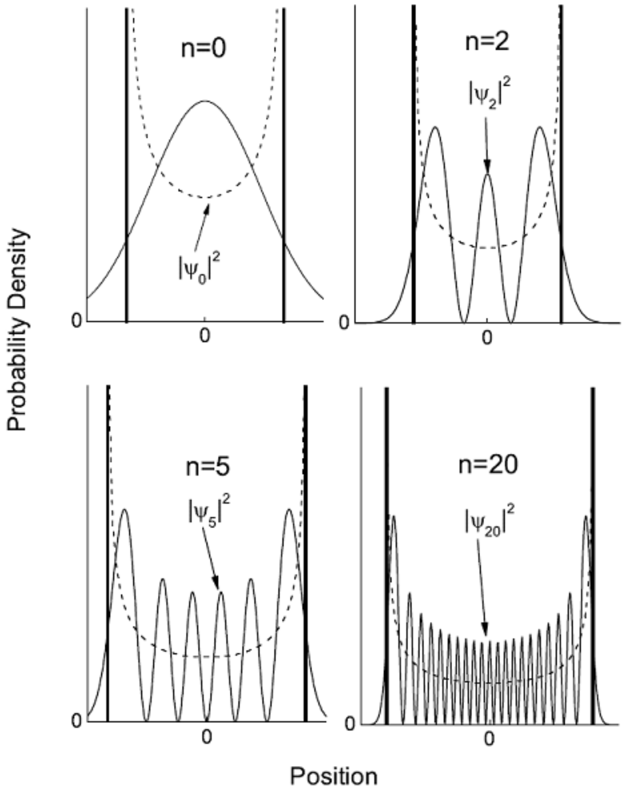

Figure 2: Probability densities for quantum and classical harmonic oscillators. At large energies, the probability density of the quantum harmonic oscillator will approach a classical regime [2].

A Statistical Mechanics and Origin of ZPE

The original basis for ZPE was conceived by Max Planck in 1911 with his theoretical model for blackbody radiation [3]. Planck made a huge intellectual leap and assumed that the energy oscillations for blackbody radiators was limited to a set of integer multiples of a fundamental unit of energy.

Planck used this merely as a mathematical device to derive a single expression for the blackbody spectrum

$$\rho(\omega) = \frac{\hbar\omega^3}{\pi^2 c^3}\left(\frac{1}{e^{\frac{\hbar\omega}{k_B T}} - 1}\right) \tag{7}$$

which matched empirical data at all wavelengths, however his idea had profound physical consequences, as well as philosophical.

In 1913 Einstein and Stern took Planck's idea a step further and hypothesized the existence of a residual energy that all oscillators have at absolute zero [3]. By studying the interaction of matter with radiation using classical physics, and developing a model of dipole oscillators to represent charged particles, Einstein and Stern modified Planck's equation using

$$\rho(\omega) = \frac{\hbar\omega^3}{\pi^2 c^3}\left(\frac{1}{e^{\frac{\hbar\omega}{k_B T}} - 1} + \frac{\hbar\omega}{2}\right) \tag{8}$$

Ultimately, Planck's assumption of energy quantization and Einstein's hypothesis became the fundamental basis for the development of quantum mechanics and began the quest for ZPE.

B The Casimir Effect and Cosmology

In 1948, Hendrik Casimir postulated that a very small force existed due the quantized nature and fluctuations of the zero-point field in a vacuum. It wasn't until the mid nineties when experiments were carried out by Steve Lamoreaux and Robert Forward to verify this effect [4].

Casimir's original goal was to quantize the van der Waals force between polarizable molecules on metal plates, however he found that the van der Waals force dropped off unexpectedly at long range separation of atoms. Instead, quantum electrodynamics predicted the existence of virtual particles which produce a non-classical force between two uncharged metal plates in close proximity.

Effectively, these particles are virtual in the sense that they "borrow" energy so long as they "return" back their energy within the smallest time length of the physical universe, i.e. as long as the energy borrowed multiplied by the time the particle exists is less than Planck's constant. In a certain sense, these particles can be understood to be a manifestation of the time-energy uncertainty principle in a vacuum [5].

Quantum electrodynamics can show how virtual photons generate a net force, and by taking two uncharged metal plates that are micrometers apart, one can show how this force interacts with the plates themselves.

The resulting Casimir force per unit area can be written

$$\frac{F}{A} = P = -\frac{dr}{da} = -\frac{\hbar c \pi^2}{240 a^4} \tag{9}$$

where r is the area between two conducting plates and a is the distance between them. This force exists in a vacuum between any pair of conductors and can be interpreted as the zero-point pressure of electromagnetic waves.

Some cosmologists have used this quantum mechanical property of ZPE in a vacuum to explain dark matter and the accelerated expansion of the universe due to its negative vacuum pressure, where the critical energy density of the accelerating expansion of the universe is on the order of 10^{-29} g/cm^3. The zero-point energy density, however, is on the order of 10^{120} g/cm^3, an astronomical discrepancy and sufficient evidence that ZPE alone does not account for this universal expansion [6].

III. CONCLUSION

The essence of Planck's zero-point energy is integral to quantum mechanics as a theory, and the implications of ZPE are endless as they are unknown. While physical evidence for ZPE has been found in phenomenon like the Casimir effect, the existence of such a force is inconclusive even on a mathematical basis. The cosmological energy density discrepancy is an unsettling problem for modern physics, and it is obvious that the true nature of ZPE has yet to be determined [1].

While the original concept of ZPE originated with Planck, Nernst, Einstein, and Stern, the original foundation for ZPE has been reworked and rethought over the past century and continues to be developed today.

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