PHYS 631: General Relativity

Homework #1

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1. **(Geometrized Units)** Express each of the pollowing quantities in two ways: i) in m^n, as meters raised to some appropriate power, and ii) in kg^n as kilograms raised to the appropriate power.

   (a) The momentum of an electron moving at $0.8c$.
   Solution:
   The gamma factor γ is

   $$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - .8^2}} = 1.67$$

   The mass of electron is $m_e = 1.21 \times 10^{-31}$kg. So th momentum is

   $$p = mv\gamma = 9.1 \times 10^{-31} \cdot 0.8 \cdot 1.67 = 1.21 \times 10^{-30} kg$$

   Since the conversion factor is $1m = 1.35 \times 10^{27} kg$ we get

   $$p = 1.21 \times 10^{-30} \left(1.35 \times 10^{27}\right)^{-1} = 8.96 \times 10^{-58} m$$

   These are the required values of momentum in each unit. □

   (b) The age of universe (13.8)Gy
   Solution:
   The age(A) in seconds is

   $$A = 13.8 \times 10^9 \cdot 365 \cdot 24 \cdot 60 \cdot 60 = 4.35 \times 10^{17} s$$

   The conversion factor is $1s = 3 \times 10^8$m so we get

   $$A = 4.35 \times 10^{17} \cdot 3 \times 10^8 = 1.3 \times 10^{26} m$$

   Since the conversion factor is $1m = 1.35 \times 10^{27} kg$ we get

   $$A = 1.3 \times 10^{26} \cdot 1.35 \times 10^{27} = 1.74 \times 10^{53} kg$$

   These are the required values. □

   (c) The orbital speed of the earth.
   Solution:
   The mass of Earth is $M = 6 \times 10^{24} kg$ which with the conversion factor $1m = 1.35 \times 10^{27}$kg becomes $M = 4.45 \times 10^{-3} m$ and the readius of earth (R) is $R = 6.4 \times 10^6 m$ and for our units $G = 1$ The orbital speed (v) is given by

   $$v^2 = \frac{GM}{R} = \frac{4.45 \times 10^{-3} m}{6.4 \times 10^6 m} = 6.97 \times 10^{-10} m^0$$
   $$v = 2.64 \times 10^{-5} m^0$$

Since the orbital speed is dimensionless, it has to have same value in kg unit also so

$$v = 2.64 \times 10^{-5} kg^0$$

These are the required values for orbital speed in each units. □

2. **(Schutz 1.3)** Draw t and x axes of the spacetime coordinates of an observer $\mathcal{O}$ and then draw:
   (a) The world line $\mathcal{O}$'s clock at $x = 1$m.
   (b) The world line of a particle moving with velocity $\frac{dx}{dt} = 0.1$, and which is at $x = 0.5$m and when $t = 0$.
   (c) The $\bar{t}$ and $\bar{x}$ axes of an observer $\bar{\mathcal{O}}$ whio moves with velocity $v = 0.5$ in the positive x direction relative to $\mathcal{O}$ and whose origin $\bar{x} = \bar{t} = 0$ coincides with that of $\mathcal{O}$.
   (d) The locus of events whose interval Δs^2 from origin is $-1m^2$.
   (e) The locus of events whose interval Δs^2 from origin is $+1m^2$.
   (f) The calibration ticks at one meter intervals along the $\bar{x}$ and $\bar{t}$ axes.

3. **(Schutz 2.1)** Given the numbers $\{A^0 = 5, A^1 = 0, A^2 = -1, A^3 = -6\}$, $\{B_0 = 0, B_1 = -2, B_2 = 4, B_3 = 0\}$, $\{C_{00} = 1, C_{01} = 0, C_{03} = 3, C_{30} = -1, C_{10} = 6, C_{11} = -2, C_{12} = -2, C_{13} = 0, C_{21} = 5, C_{22} = 2, C_{23} = -2, C_{20} = 4, C_{32} = -1, C_{32} = -3, C_{33} = 0\}$, find:
   (a) $A^\alpha B_\alpha$
   Solution:

$$A^\alpha B_\alpha = 5 * 0 + 0 * -2 + -1 * 4 + 6 * 0 = -4$$

□

   (b) $A^\alpha C_{\alpha\beta}$ for all β
   Solution:
   for $\beta = 0$

$$\begin{aligned} A^\alpha C_{\alpha 0} &= A^0 C_{00} + A^1 C_{10} + A^2 C_{20} + A^3 C_{30} \\ &= 5 * 1 + 0 * 5 + -1 * 4 - 6 * -1 = 7 \end{aligned}$$

   Similarly

$$\begin{aligned} A^\alpha C_{\alpha 1} &= 0 + 0 + -5 + 6 = 1 \\ A^\alpha C_{\alpha 2} &= 10 + 0 + -2 + 18 = 26 \\ A^\alpha C_{\alpha 3} &= 15 + 0 + 3 + 0 = 18 \end{aligned}$$

□

   (c) $A^\gamma C_{\gamma\sigma}$ for all σ
   Solution:
   This is same as the previous one because the dummy index is the only one different. □

   (d) $A^\nu C_{\mu\nu}$ for all μ
   Solution:

□

   (e) $A^\alpha B_\beta$ for all α, β
   (f) $A^i B_i$
   (g) $A^j B_k$ for all j, k

4. **(Schutz 2.14)** The following matrix gives a Lorents transformation from $\mathcal{O}$ to $\bar{\mathcal{O}}$:

$$\begin{bmatrix} 1.25 & 0 & 0 & 0.75 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0.75 & 0 & 0 & 1.25 \end{bmatrix}$$

(a) What is the velocity of $\bar{\mathcal{O}}$ relative to $\mathcal{O}$?
(b) What is the inverse matrix to the given one?
(c) Find the components in $\mathcal{O}$ of a vector $\boldsymbol{A} \to (1, 2, 0, 0)$.

5. **(Schutz 2.22)**
   (a) Find the energy, rest mass and three-veloity v of a particle whose four momentus has the components $(0, 1, 1, 0)kg$.
   (b) The collision of two particles of four-momentum

$$\boldsymbol{p}_1 \underset{\mathcal{O}}{\longrightarrow} (3, -1, 0, 0)kg, \qquad \boldsymbol{p}_2 \underset{\mathcal{O}}{\longrightarrow} (2, 1, 1, 0)kg$$

   results in the destruction fo the two particle and the production fo three new ones, two of which have four-mementa

$$\boldsymbol{p}_3 \underset{\mathcal{O}}{\longrightarrow} (1, 1, 0, 0)kg, \qquad \boldsymbol{p}_4 \underset{\mathcal{O}}{\longrightarrow} (1, -1/2, 0, 0)kg$$

   Find the four-meomentum, energy, rest mass and three velocity of the third particle produced. Find the CM frame's three-velocity.

6. **(Schutz 2.30)** The four-velocity of a rocket ship is $\boldsymbol{U} \underset{\mathcal{O}}{\longrightarrow} (2, 1, 1, 1)$. It encounters a high-velocity cosmic ray whose mementum is $\boldsymbol{P} \underset{\mathcal{O}}{\longrightarrow} (300, 299, 0, 0) \times 10^{-27}kg$. Compute the energy of the cosmic ray as measured by the rocket ship's passengers, using each of the two following methods.
   (a) Find the Lorentz transformation from $\mathcal{O}$ to the MCRF of the rocket ship, and use it to tranform the componetns of $\boldsymbol{P}$.
   (b) Use eq 2.35
   (c) Which method is quicker? Why?