PHYS 576: Particle Physics

Homework #1

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1. **(Griffith 1.2)** The mass of Yukawa's meson can be estimated as follows. When two protons in a nucleus exchange a meson (mass m), they must temporarily violate the conservation of energy by an amount mc^2(the rest energy of the meson). The Heisenberg uncertainty principle says that you may 'borrow' an energy ΔE, provided you 'pay it back' in a time Δt given by $\Delta E \Delta t = \frac{\hbar}{2}$ (where $\hbar = h/2\pi$). In this case, we need to borrow $\Delta E = mc^2$ long enough for the meson to make it from one proton to the other. It has to cross the nucleus (size r_0), and it travels, presumably, at some substantial fraction of the peed of light, so , roughly speaking, $\Delta t = \frac{r_0}{c}$. Putting all this together, we have

$$m = \frac{\hbar}{2r_0 c}$$

Using $r_0 = 1 \times 10^{-13}$cm, calculate the mass of Yukuwa's meson. Express your answer in $\frac{MeV}{c^2}$, and compare the observed mass of πon.
Solution:
Given $r_0 = 1 \times 10^{-15} m$, $M = 6.58 \times 10^{-22} MeV s; c = 3 \times 10^8 s$ we can substitute to find the total mass

$$m = \frac{\hbar}{2r_0 c} = \left(\frac{\hbar c}{2r_0} \frac{1}{c^2} \right) = 98.7 \frac{MeV}{c^2}$$

So the predicted mass is $98.7 MeV$, but the real mass of Yukuwa's meson is $138 Mev$ which is off by a factor of about 1.4. □

2. (a) Members of baryon decuplet typically decay after 1×10^{-23} seconds into a lighter baryon (from the baryon octet) and a meson (from the pseudo-scalar meson octet). Thus for example, $\Delta^{++} \to p^+ + \pi^+$. List all decay methods of this form for the Δ^-, Σ^{*+} and Ξ^{*-}. Remember that these decays must conserve charge and strangeness(they are strong interactions).
Solution:
The decay has to satisfy the charge conservation and strangeness conservation. The possible decay for each of these are:

$$\begin{aligned}
&\Delta^- \to n + \pi^- \text{ and } \Sigma^- + K^0 \\
&\Sigma^{*+} \to p + \bar{k}^0; \quad \Sigma^+ + \pi^0; \quad \Sigma^+ + \eta; \quad \pi^0 + \Sigma^0; \quad \Lambda + \pi^+; \quad K^+ + \Xi^0 \\
&\Xi^{*-} \to \Sigma^0 + K^-; \quad \Xi^- + \pi^0; \quad \Sigma^- + \bar{K}^0; \quad \Lambda + K^-; \quad \Xi^0 + \pi^-; \quad \Xi^- + \eta
\end{aligned}$$

These are all the possible decay schemes that preserve charge and strangeness. □

(b) In any decay, there must be sufficient mass in the original particle to cover the masses of the decay products. (There may be more than enough; the extra will be 'soaked up' in the form of kinetic energy int the final state.) Check each of the decay you proposed in part (2) to see which ones meet this criterion. The others are kinematically forbidden.
Solution:
Each of these decays are two body decays of the form $A \to B + C$, the threshold energies in each can

be calculated with

$$E = \frac{M^2 - m_B^2 - m_c^2}{2M_A}$$

Using the mass value of each of these products we find that the only allowed decays are

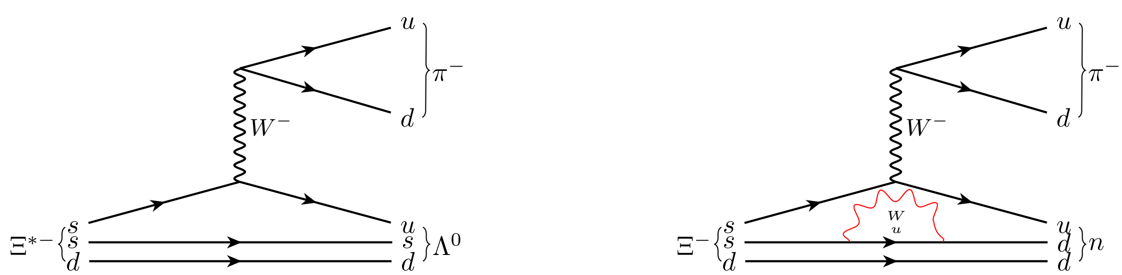
$$\begin{aligned}
&\Delta^- \to \pi^- + n \\
&\Sigma^{*+} \to \Sigma^+ + \pi^0; \qquad \Lambda + \pi^+; \qquad \Sigma^0 + \pi^- \\
&\Xi^{*-} \to \Sigma^0 + \pi^-; \qquad \Xi^- + \pi^0
\end{aligned}$$

These are the only allowed decays. $\square$

3. **(Griffith 2.5)**
   (a) Which decay do you think would be more likely,

$$\Xi^- \to \Lambda + \pi^- \qquad \text{or} \qquad \Xi^- \to n + \pi^-$$

Solution:
Although the decay $\Xi^- \to n + \pi^-$ is favored kinematically over the decay $\Xi^- \to \Lambda + \pi^-$ strangeness



(a) $\Xi^{*-} \to \pi^- + \Lambda$ (b) $\Xi^{*-} \to \pi^- + n$

Figure 1: Feynman diagram for two different decays.

conservation favors the second one. Since the two s quarks have to be conserved (strangeness conservation); an extra W^- is requires. This means there are two extra weak vertices. Higher number of vertices would make the process much more less likely. $\square$

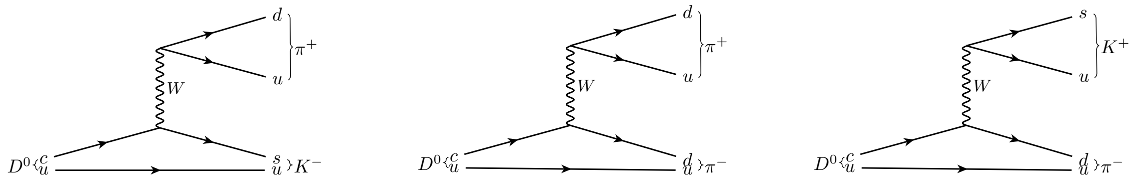
   (b) Which decay of $D^0(c\bar{u})$ meson is most likely

$$D^0 \to K^- + \pi^+ \text{ or } D^0 \to \pi^- + \pi^+, \text{ or } D^0 \to K^+ + \pi^-$$

Which is least likely? Draw the Feynman diagrams, explain your answer and check the experimental data.
Solution:
The Feynman diagram for $D^0 \to K^- + \pi^+$ is $\square$
The second decay is more favored because there is no generation cross over in the particle decay. When there is a generation cross over in the decay process it is less favored in the decay although it is allowed kinematically. So the most favored decay process is $D^0 \to \pi^- + \pi^+$.

4. **(Griffith 3.13)** Is p^μ timelike, spacelike, or lightlike for a (real) particle of mass m ? How about a massless particle? How about a virtual particle?
Solution:
To determine the nature of the particles we find the Lorentz scalar for each. Finding $p^2 = p \cdot p = p^\mu p_\mu$ we get

$$p^2 = m^2 c^2$$



(a) $D^0 \to \pi^+ + K^-$ (b) $D^0 \to \pi^- + \pi^+$ (c) $D^0 \to \pi^- + K^+$

Figure 2: Feynman diagram for three different decay schemes for D^0.

For a real particle with mass m the quantity $p^2 > 0$ so the particle is timelike. For a massless particle γ the scalar $p^2 = 0$ so this is lightlike. And for virtual particle the nature depends upon the mass as there could be massless and massive virtual particles. $\square$

5. **(Griffith 3.16)** Particle A(Energy E) hits particle B (at rest), producing $C_1, C_2, \ldots A + B \to C_1 + C_2 + \ldots C_n$. Calculate the threshold(i.e., minimum E) for this reaction, in terms of various particle masses.
Solution:
In the lab frame lets consider particle A with mass m_A and momentum $\vec{p}_A$ with energy E strikes a stationary target particle B with mass m_B. The four momentum of A is $p^\mu_A = (E, \vec{p}_A)$ and the four momentum of B is $p^\mu_B = (m_b, 0)$. The invariant Lorentz scalar in the lab frame is

$$p^2 = (p^\mu_A + p^\mu_b)^2 = (E + m_B, \vec{p}_A)^2 = E^2 + m_B^2 + 2Em_B - |\vec{p}_A|^2$$

But for particle A we have $E^2 - |\vec{p}_A|^2 = m_A^2$ substituting this in above expression we get

$$p^2 = m_A^2 + m_B^2 + 2Em_B$$

Since this Lorentz scalar is invariant in any reference frame we have to have the same value for the p^2 for the final products. For threshold condition the daughter particles are just created so thy do not carry any momentum. Which implies for each particles their momentum $m_n = E_n$ so for each of them the four momentum is $p^\mu_n = (m_n, 0)$. The Lorentz scalar for the final qty is

$$p^2 = (p^\mu_1 + p^\mu_2 + p^\mu_n)^2 = (m_1 + m_2 + \ldots + m_n, 0)^2 = (m_1 + m_2 + \ldots + m_n)^2 - 0 = M^2 \text{ (say)}$$

where the symbols M is used to mean the total sum of masses of all daughter particles. Equating the Lorentz scalar we get

$$M^2 = m_A^2 + m_B^2 + 2Em_b \qquad \Longrightarrow \; E = \frac{M^2 - m_A^2 + m_B^2}{2m_B}$$

This gives the threshold energy in lab frame of the incoming particle. $\square$

6. **(Griffith 3.22)** Particle A, at rest, decays into three or more particles: $A \to B + C + D + \ldots$
   (a) Determine the maximum and minimum energies that B can have in such a decay, in terms of the various masses.
   Solution:
   The mimimum energy for the outgoing particle is equal to its mass when the produced particle is just created and has no spatial momentum and all other energy is carried away by the other outgoing particles.

$$E_{\min} = m_B$$

   The maximum energy is carried by particle B when the particle A decays in such a way that particle B moves in one direction and all other particles move in other direction in unison. Since we would get maximum energy when the other particles do not move relative to each other giving maximum energy, this implies that all other particle move as a single unit of total mass with the sum of their masses. So we can rewrite the decy as

$$A \to B + (C + D \dots) \equiv B + N$$

where the particle N is as it its a single particle with the mass equal to sum of masses of each of the rest of daughter particles.

$$m_N = m_c + m_D + \dots$$

This problem is now like a single particle decaying into two with equal and opposite momentum. In the CM frame the value of Lorentz scalar $p^2 = M_A^2$

$$\begin{aligned} p_A^\mu = p_B^\mu + p_N^\mu &\equiv (m_A, 0) = (E_B, \vec{p}_B) + (E_N, -\vec{p}_B) \\ &\implies (E_N, -\vec{p}_B) = (m_A, 0) - (E_B, \vec{p}_B) \end{aligned}$$

Squaring both sides and equating

$$\begin{aligned} (E_N, -\vec{p}_B)^2 &= (m_A, 0)^2 + (E_B, -\vec{p}_B)^2 - 2(m_A, 0) \cdot (E_B, \vec{p}_B) \\ E_N^2 - |\vec{p}_B|^2 &= m_A^2 + E_B^2 - |\vec{p}_B|^2 - 2m_A E_B \end{aligned}$$

Since we have $m^2 = E^2 - |\vec{p}|^2$ we get

$$\begin{aligned} m_N^2 &= m_A^2 + m_B^2 - 2m_A E_B \\ 2m_A E_B &= m_A^2 + m_B^2 - m_N^2 \\ E_B &= \frac{m_A^2 + m_B^2 - (m_C + m_D + \dots)^2}{2m_A} \end{aligned}$$

This gives the maximum energy of the particle B. $\square$

(b) Find the maximum and minimum electron energies in muon decay, $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$.
Solution:
The minimum energy of the electron is the mass of electron itself (in natural units of course) so

$$E_{\min} = m_e = 511 keV$$

By above discussion the maximum energy is

$$E_{\max} = \frac{m_\mu^2 + m_{e^-}^2 - (m_{\mu_e} + m_{\bar{\mu}_e})^2}{2m_\mu}$$

Since the neutrinos have very tiny mass (almost massless) we ignore their masses sow we have

$$E_{\max} \approx \frac{105^2 - 0.511^2}{2 \times 105^2} = 52.50 MeV$$

This gives the maximum mass of the outgoing muon. $\square$