PHYS 511: Electrodynamics

Homework #6

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1. **(Jackson 4.12)** Water vapor is a polar gas whose dielectric ocnstant exhibits and appreciable temperature dependence. The following table gives experimental data on this effect. Assuming that water vapor obeys the ideal gas law, calculate the molecular polarizability as a function of innverse temperature and plot it. From the solope of the curve, deduce a value for the permanent dipolemomont of the H_2O molecule.

T(K)	Pressure (cm Hg)	$(\frac{\epsilon}{\epsilon_0} - 1) \times 10^5$
393	56.49	400.2
423	60.93	371.7
453	65.34	348.8
483	69.75	328.7

Solution:
With the ideal gas equation we have

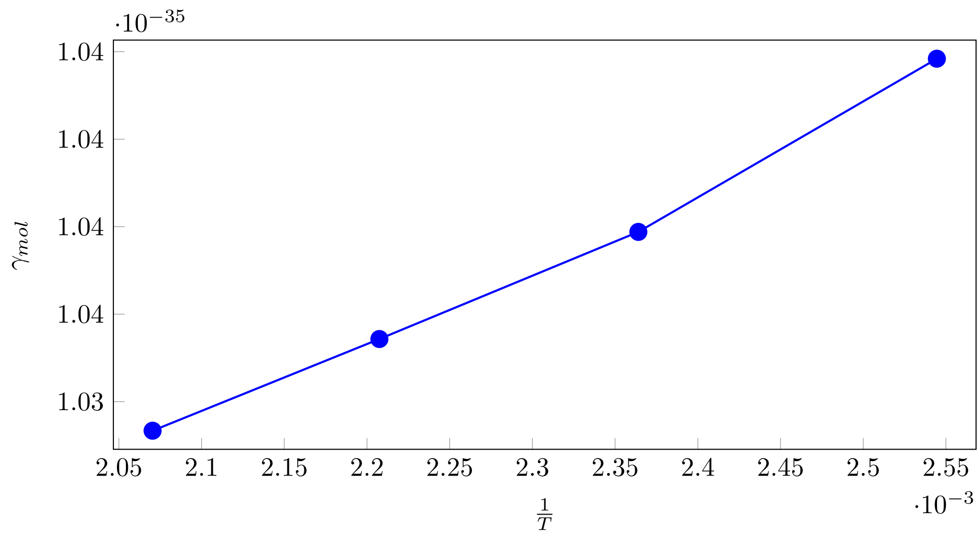
$$PV = NkT \qquad \Longrightarrow n = \frac{N}{V} = \frac{P}{kT}$$

By Clausis-Mossetti equation we have the molecular polarizability is given by

$$\gamma = \frac{3}{n}\left(\frac{\epsilon/\epsilon_0 - 1}{\epsilon/\epsilon_0 - 2}\right) = \frac{3kT}{P}\left(\frac{\epsilon/\epsilon_0 - 1}{\epsilon/\epsilon_0 - 2}\right)$$

Plotting this as a function of $\frac{1}{T}$ gives The slope is 8.9×10^{-35}

□



2. **(Jackson 4.13)** Two long, coaxial, cylindrical conductin surfaces of radii a and b are lowered vertically into a liquid dielectric. If the liquid raises an average height h between the electrodes when a potential difference V is established between them, show that th esusceptibility of the liquid is

$$\chi_e = \frac{b^2 - a^2 \rho g h \ln(b/a)}{\epsilon_0 V^2}$$

where ρ is the density of the liquid, g is the accleration due to gravity, and the susceptibility of th air is neglected.
Solution:
The total energy in the capacitor of capacitance C is given by

$$E = \frac{1}{2} C V^2$$

The capacitance of coaxial cylinder per unit length is given by

$$C = \frac{2\pi\epsilon_0}{\ln(b/a)}$$

Let l is the length of the cylindrical conductors above the liquid, of which the liquid raises upto h. The section with $l - h$ is air filled and the section with height h above the liquid surface is dielectric filled. So the capacitance of each section gives

$$C_{air} = \frac{2\pi\epsilon_0 (l-h)}{\ln(b/a)}; \qquad C_{\text{liquid}} = \frac{2\pi\epsilon h}{\ln(b/a)}$$

The total upward force on the raised liquid is thus

$$F = \frac{\mathrm{d}E}{\mathrm{d}h} = \frac{1}{2} V^2 \frac{\mathrm{d}C}{\mathrm{d}h} = \frac{1}{2} V^2 \frac{\mathrm{d}}{\mathrm{d}h} \left(\frac{2\pi\epsilon_0 (l-h)}{\ln(b/a)} + \frac{2\pi\epsilon h}{\ln(b/a)} \right)$$
$$= \frac{\pi}{\ln(b/a)} \left[-\epsilon_0 + \epsilon \right]$$

But we have $\epsilon = \epsilon_0 + \chi_e \epsilon_0$, so we get

$$F = \frac{\pi \chi_e \epsilon_0}{\ln(b/a)}$$

This force is balanced by the gravitational force in quilibrium which is given by

$$F = mg = \rho V_r g$$

The volume of raised liquied V_r is

$$V_r = \pi(b^2 - a^2) h$$

Thus

$$F = \rho \pi (b^2 - a^2) h g$$

Equating the forces

$$\frac{\pi \chi_e \epsilon_0}{\ln(b/a)} = \rho \pi (b^2 - a^2) h g$$
$$\chi_e = \frac{(b^2 - a^2) \rho g h \ln(b/a)}{\epsilon_0 V^2}$$

This is the required expression. □