Detecting Earth-Sized Planets with Laser Frequency Combs

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Abstract

Detection of Earth-mass exoplanets from the Doppler shift of stellar spectral lines is limited by the range and resolution of available reference spectra. Laser frequency combs are shown to produce a spectra of evenly-spaced lines spanning the optical into the infrared. Options for calibrating the comb and determining line positions are explained. The line spacing of laser frequency combs are adjusted to work with the resolution of astronomical spectrographs. Radial velocity measurements from stellar spectra calibrated with a laser frequency comb are anticipated to have a precision on the order of 1 cm/s. Laser frequency combs are presented as the cutting edge of technology that will make it possible to discover Earth-mass exoplanets.

1. Introduction

Extrasolar planets are most commonly observed indirectly. The gravitational attraction between a star and its planet will cause both bodies to orbit about the system's center of mass, which is often still inside the radius of the star. Nonetheless, the wavelength of spectral emission lines from the star decreases as the star moves toward the observer and increases as it moves away. The star's radial velocity is measured from the Doppler shift of its spectral lines. When a series of spectra are taken over time, the existence of an extrasolar planet can be inferred from periodic variations in the parent star's radial velocity. Whether it is possible to notice these small variations depends on the resolution of the spectra.

Detecting rocky, Earth-mass planets in stellar habitable zones is a challenge. A Jupiter-mass planet at a particular radius exerts a stronger gravitational force and causes a larger radial velocity change than an Earth-mass planet. The Earth shifts the Sun's radial velocity by just ± 0.1 m/s per year. Even at its larger radius, each orbit of Jupiter causes a change of ± 13 m/s. Doppler wobble from the spectra of a parent star is more noticeable for more massive planets. The precision required to observe periodic Doppler shifts in stellar spectra is the main limitation for finding Earth-mass planets around solar mass stars.

The range of radii within which a planet could sustain liquid water and Earth-like life changes based on the size of the star. M stars, which are less than half the mass of the Sun, have habitable zones that lie closer in. Searching for Earth-mass planets becomes more practical with M stars as candidates. An Earth-mass planet orbiting an M star causes a larger velocity shift (2 m/s) than the Earth does orbiting the Sun. The smaller radius means Earth-mass planets orbiting M stars have shorter orbital periods (~3 days). Variability data can be collected much faster than waiting a full year for an Earth-mass planet to orbit a solar mass star. This improved detectability and their abundance in the solar neighborhood make M stars good targets for exoplant searches. The drawback is that M stars are brighter in the infrared than in the optical, due to their cooler temperatures. Infrared wavelengths lack reference spectra to compare with the stellar spectra.

Laser frequency combs provide the precision necessary to detect radial velocity shifts on the order of 1 cm/s and solve the problem of not having reference spectra in the infrared. Section 2 explains how the position of laser frequency comb lines is determined, calibrated, and manipulated. Section 3 shows how the precise, even spacing of the laser frequency comb can be exploited to calibrate stellar spectra, and discusses the anticipated velocity precision. Section 4 summarizes how using laser frequency combs as calibration spectra remedies two main technical challenges of detecting Earth-mass exoplanets.

2. Laser Frequency Combs

Laser frequency combs get their name from the regularly-spaced spike pattern that appears when pulses of light emitted by a laser are Fourier transformed from the time domain to the frequency domain. The position of lines in a laser frequency comb spectrum depend on the repetition and carrier wave frequencies, which can be locked to an atomic clock. The extreme precision of laser frequency comb spectra makes it possible to take previously unattainable measurements.

2.1 Evenly-Spaced Modes

The laser emits pulses of light that have a periodic group velocity. Each pulse is separated by the round trip time for light reflecting in the laser cavity. The round time time is twice the length of the cavity divided by the group velocity:

$$T = 2\mathrm{L}/v_g$$

The pulse envelope travels at the group velocity. However, the carrier wave has a phase velocity that shifts by an amount Δφ with every round trip through the cavity (Figure 1).

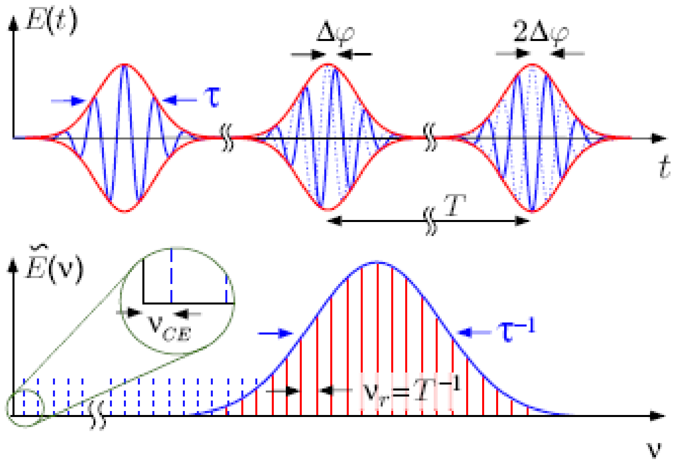

Figure 1: Fourier transform of laser light

(Figure: Murphy et al., 2007)

A periodic pulse generates peaks in frequency space that are exactly the width of the repetition frequency

$$f_{rep}=\frac{1}{T}=\frac{v_g}{2\mathrm{L}}$$

However, the shift in the carrier wave phase velocity causes peak positions to be offset from exact multiples of the repetition frequency. This offset Δφ is given by

$$\Delta\varphi=2\pi f_c T$$

where f_c is the carrier wave frequency. The carrier wave frequency influences the location of the evenly-spaced peaks in frequency space. The previous equation can be manipulated to show that it is

$$f_c=\frac{\Delta\varphi}{2\pi T}=\frac{\Delta\varphi f_{rep}}{2\pi}$$

The position of each line in the laser frequency comb is determined by the repetition frequency offset and the carrier wave frequency.

The pulse envelope traveling at v_g has a periodic envelope function

$$\mathrm{A(t) = A(t\text{-}T)}$$

that describes the carrier wave amplitude. The electric field outside the cavity can therefore be written

$$E(t)=A(t)\mathrm{e}^{-i2\pi f_c t}=\sum_n A_n e^{-i2\pi(nf_{rep}+f_c)t}$$

Due to the phase shift in the time domain, the nth frequency domain peak is at

$$f_n = n f_{rep} + f_c$$

The repetition frequency sets the spacing of the comb lines, and the carrier wave frequency offsets their positions from exact multiples of the repetition frequency. The laser must be mode-locked so that Δφ and f_{rep} do not drift as the laser operates. If these parameters were allowed to vary, then the exact position of comb lines would not be known.

2.2 Calibration

The repetition frequency and the carrier wave frequency must both be determined to know the frequency of each tooth in the comb. A photodetector can measure the repetition frequency from the beam, and the carrier wave frequency is obtained from self-referencing the comb (Figure 2). A nonlinear fiber optic cable extends the frequency range of the comb so that it spans an optical octave. A low frequency mode ***n*** is doubled and compared with the ***2n*** high frequency mode. The frequency-doubled mode is superimposed on the high frequency mode for a photodetector to read. The carrier frequency is the difference between frequency-doubled ***n*** and ***2n***.

$$2(nf_{rep} + f_c) - (2\mathrm{nf}_{rep} + f_c) = f_c$$

Self-referencing different pairs of modes can be used to verify consistency across the frequency range.

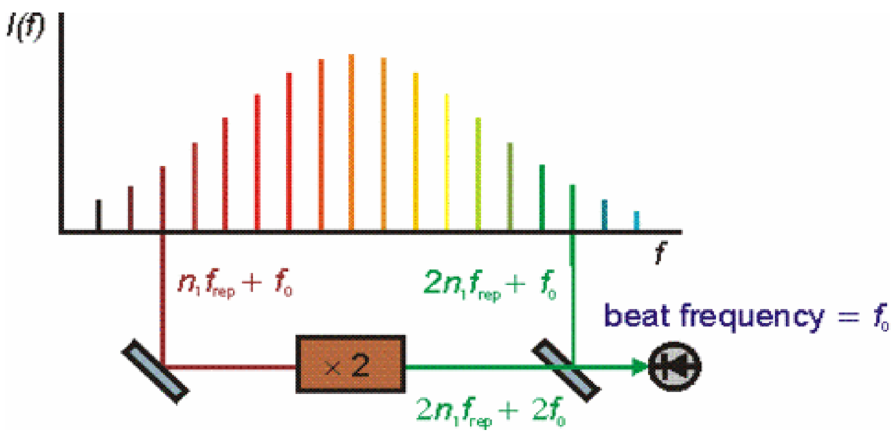

Figure 2: Comparision of frequency-doubled mode n with 2n mode

(Figure: National Physics Laboratory website)

Alternatively, the frequency comb can be calibrated by referencing one line to a continuous wave single-frequency laser. Once the frequency of one line is known, all the rest are determined by adding integer multiples of the repetition frequency. For this calibration technique, it is not necessary to know the carrier wave frequency. The continuous wave single-frequency laser introduces additional uncertainty, so self-referencing the laser frequency comb is more precise.

2.3 Controlling Line Spacing

Once the laser frequency comb has been created and calibrated, the line spacing can be widened by piping the light pulses through a Fabry-Perot cavity. Light in the Fabry-Perot cavity reflects off of two mirrors a distance L apart. The free spectral range depends on the cavity length

$$FSR=\frac{c}{2\mathrm{L}}$$

A comb line will be transmitted if it is a multiple (m) of the free spectral range.

$$m f_{rep}=FSR=\frac{c}{2\mathrm{L}}$$

Lines that are not integer multiples of the free spectral range are suppressed. The Fabry-Perot cavity thins out the comb lines without changing their positions, as long as the free spectral range is an integer multiple of the repetition frequency.

3. Application to Spectroscopy

Laser frequency combs are currently being tested at small observatories with high-resolution spectrographs. Once the instrumentation works well together, the laser frequency comb will allow precise measurements of stellar spectra at the optical and infrared wavelengths. It will be possible to measure radial velocity shifts on the order of 1 cm/s and detect Earth-mass planets around solar mass stars. From the Doppler shift equation, the necessary fractional accuracy is

$$\frac{\Delta\lambda}{\lambda}=\frac{v}{c}=\frac{0.1\,m/s}{3\mathrm{x}10^{8}\,m/s}=3.33\mathrm{x}10^{-11}$$

Frequency combs from Ti:Sapphire lasers achieve a fractional accuracy better than 10^{-12} by synchronizing the repetition and carrier wave frequencies with a highly accurate time source, like the Global Positioning System. Instrumental effects from interfacing the laser frequency comb with a spectrograph contribute uncertainties that decrease precision.

3.1 Apparatus

When a laser frequency comb is used to calibrate stellar spectra, the laser output is transmitted to the spectrograph with an optical fiber (Figure 3). The repetition rate of the laser that generates the frequency comb is mode-locked to an atomic clock for stability. The resulting frequency comb spectrum is calibrated against a continuous wave laser. The frequency comb then passes through a Fabry Perot cavity to increase the spacing between comb lines. A lock-in amplifier stabilizes the output of the Fabry-Perot cavity. The thinned-out laser frequency comb spectrum is then recorded next to the stellar spectrum on the CCD (Figure 4).

Having more teeth in the frequency comb improves the precision of the wavelength calibration. However, the spectrograph must be able to resolve the line spacing of the laser frequency comb for the calibration spectra to be useful. With an optimal line spacing, the teeth are frequent enough to provide a precise calibration, but sparse enough to preserve the contrast between neighboring lines.

3.2 Observatories and Resolution Capabilities

Laser frequency combs are produced at observatories using either 250 MHz Er:fiber or 1 GHz Ti:Sapphire lasers. The German Vacuum Tower Telescope has taken spectra of the Sun's photosphere in the infrared with the laser frequency comb from an Er:fiber laser (Figure 4). NIST also has plans to provide the FIRST spectrograph at the Apache Point Observatory with calibration in the infrared from an Er:fiber laser frequency comb. A Ti:sapphire laser frequency comb operates at the Whipple Observatory on Mount Hopkins in Arizona as a prototype for one that will be installed on the HARPS-NEF spectrograph in Chile. Properties of the spectrographs used determine what sort of precision is available from the calibration spectra of laser frequency combs.

The error on the recorded position of the comb line is approximated by:

$$\delta_f \approx A \frac{FWHM}{S/N\,\sqrt{n}}$$

S/N is the peak Signal to Noise of the line, n is the number of pixels sampling the Full Width Half Maximum, and A is a pre-factor that characterizes noise. A spectrograph with an echelle diffraction grating separates light into multiple orders that are imaged on a CCD. Each echelle order spans 2048 pixels of the CCD, which has 3-pixel FWHM sampling for a line. Assume a comb line can occur every 2.5 resolution elements without saturating the detector. This results in the number of comb lines, N

$$N = \frac{2048\,pixels}{(3\,\frac{pixels}{res\,elem})(\frac{2.5\,res\,elem}{1\,line})} = 272\,lines$$

The average precision for the comb spectrum is then:

$$\sigma_f \approx \frac{\delta_f}{\sqrt{N}} = A \frac{FWHM}{S/N \sqrt{n} \sqrt{N}}$$

A high-resolution spectrograph with a resolving power of R = 150000 has a FWHM of 2 km/s. For the FWHM measurement, the frequency has already been converted to a velocity using the Doppler shift expression. Fitting the noise profile gives a pre-factor of A = 0.41. A velocity measurement can then be known to within

$$\sigma_v \approx \frac{(0.41)(2\times10^5 cm/s)}{(500)\sqrt{3*272}} \approx 5.7\, cm/s$$

This result is comparable to the 1 cm/s precision anticipated from the Ti:Sapphire laser frequency comb designed for the observatory in Chile. As laser frequency combs transition from the developmental stage to astrophysical applications, instrumental effects are the limiting factor for precise measurements. Optimizing the interaction of laser frequency combs with the spectrographs is the next step for achieving precise spectra to detect exoplanets.

4. Conclusions

The position of a line in a laser frequency comb is an integer multiple of the repetition frequency offset by the carrier wave frequency. The even spacing of the comb means that all line positions are known once one is determined. Synchronizing the repetition and carrier wave frequencies with an atomic clock or GPS produces calibration spectra that are accurate enough to detect a $\pm$10 cm/s change caused by an Earth-mass planet orbiting a stellar mass star. The range that a laser frequency comb can cover provides the infrared end of the spectrum with precise, abundant reference lines. Observations of M stars in the infrared are no longer constrained by lack of reference spectra. The precisions and range of laser frequency combs solve two major technical challenges for detecting exoplanets.

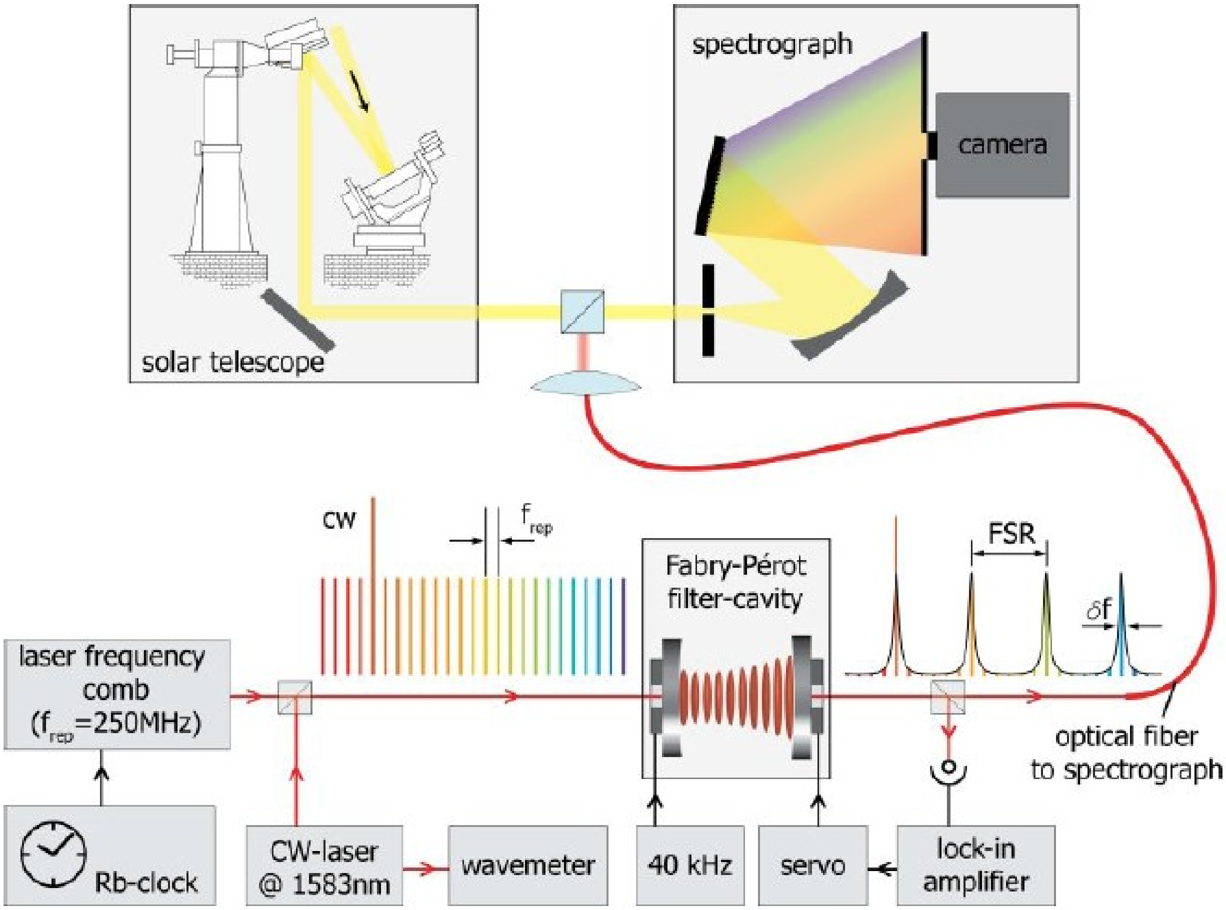

Figure 3: Vacuum Tower Telescope Er:fiber laser frequency comb apparatus diagram

(Figure: Steinmetz et al., 2008)

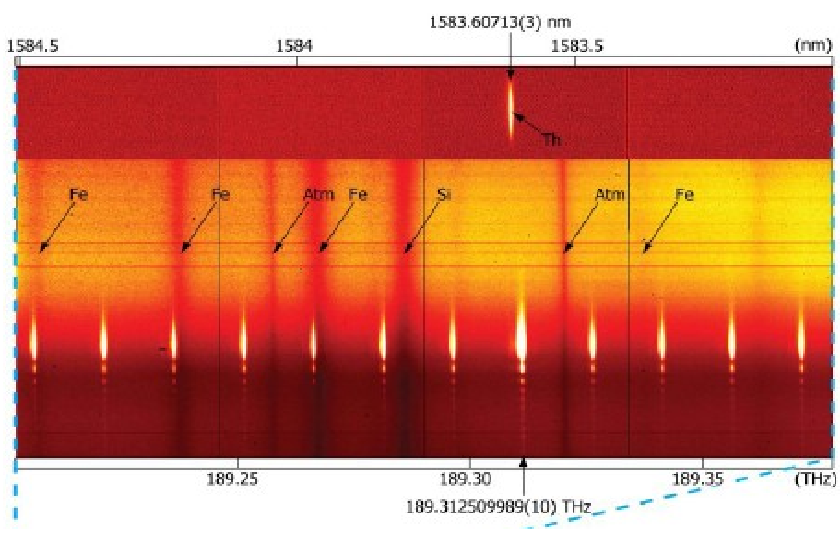

Figure 4: Laser frequency comb spectra (white) imposed on solar photosphere spectra from VTT

(Figure: Steinmetz et al., 2008)

References

1. Li, Chih-Hao et al., “A laser frequency comb that enables radial velocity measurements with a precision of 1 cm/s”, Nature, 2008, vol. 452, pp. 610-12.

2. Murphy, M et al., “High-precision wavelength calibration of astronomical spectrographs with laser frequency combs”, Monthly Notices of the Royal Astronomical Society, 2007, vol. 380 pp. 839-47.

3. Osterman, S. et al., "A Near Infrared Laser Frequency Comb for High Precision Doppler Planet Surveys”, Conference Proceedings, New Technologies for Probing the Diversity of Brown Dwarfs and Exoplanets, Shanghai, China, 19-24 July, 2009. Submitted to Eur. Phys. J. Conferences

4. Steinmetz, Tilo et al. “Laser frequency combs for astronomical observations”, Science, 2008, vol. 321, pp. 1335 – 7.

5. Udem, Th., Holzwarth, R., and Hansch, T., “Optical frequency metrology”, Nature, 2002, vol. 416, pp. 233-7.

6. Walker, Gordon, “Extrasolar planets: With a coarse-tooth comb”, Nature, 2008, vol. 452, pp. 538-9.