PHYS 105

In-class exercise 8.2

A More Realistic Treatment of Dissipation

Now let's consider the problem of launching a high-speed projectile and determining its **range**, **maximum height**, and **time of flight**, as in in-class exercise 8.1. We will investigate how those quantities change as we add more realism to the calculation.

1. Start with the general two-dimensional predictor–corrector scheme introduced previously. Consider a projectile launched with speed $v_0 = 1500$ m/s at an angle of $\theta_0 = 20°$ to the horizontal. Take gravity to be uniform,

$$\begin{aligned} a_x &= 0 \\ a_y &= -g = -9.80 \text{ m/s}^2 , \end{aligned}$$

   and use a timestep of $\delta t = 0.1$ s. Continue your calculation until the projectile strikes the ground, at $y = 0$. What are the range, maximum height, and time of flight? As usual, use linear interpolation (to $y = 0$) to refine your answer for the range and time of flight. Use `double` precision throughout.

2. Model the effect of air resistance by adding a "βv^2" term to the acceleration:

$$\begin{aligned} a_x &= -\beta v v_x \\ a_y &= -g - \beta v v_y , \end{aligned}$$

   where $v = |\mathbf{v}| = \sqrt{v_x^2 + v_y^2}$ and $\beta = 10^{-5}$.

   - By how much does the range decrease?
   - What value of v_0 is needed to restore the range in part 1 (to 1 percent accuracy)? What is the new time of flight?
   - Notice that the trajectory is quite asymmetrical, unlike the case without air resistance. Verify this quantitatively by calculating and printing out the angle θ_1 to the horizontal at which the projectile strikes the ground.