PHYS 105
In-class exercise 7.2

The driven nonlinear pendulum

Consider the motion of the nonlinear, damped, driven pendulum, described by the equation

$$a \equiv \frac{d^2x}{dt^2} = -k\sin x - \alpha v + g\cos(\omega_D t)\,,$$

(where again we have replaced θ in the earlier discussion with x).

1. Compute the motion of the system and plot `x(t)` for $x_0 = v_0 = 0$, $k = 1$, $\alpha = 0.5$, $\omega_D = 2./3$, and $g = 1.0$, 1.07, 1.1, and 1.15.

   Start each calculation at time $t = 0$, begin plotting trajectories at $t = 250$ to allow initial transients to die away, and continue each calculation to $t = 1500$.

   Can you see any similarities or differences between the four cases?

2. One complicating factor is that sometimes the pendulum "loops the loop" and the angle `x` increases without bound. Modify your program to wrap the variable `x` so that it always lies in the range $[-\pi, \pi]$, as follows:

```
while (x >  M_PI) x -= 2*M_PI;
while (x < -M_PI) x += 2*M_PI;
```

   or

```
while x >  math.pi: x -= 2*math.pi
while x < -math.pi: x += 2*math.pi
```

   You can accomplish the same result using the `fmod` functions in C++ or Python, but be careful with their treatments of negative numbers.

   Plot the *phase-space* trajectories—`v` versus wrapped `x`—of each of the calculations performed in part (1).