PHYS 105

In-class exercise 10.2

Variable Time Steps

Nothing about the predictor–corrector scheme we have been using requires us to keep the time step constant. Perform the following calculations:

(i) $r_0 = 1.0, v_0 = 0.5, \delta t = 0.01$,

(ii) $r_0 = 1.0, v_0 = 0.3, \delta t = 0.01$,

(iii) $r_0 = 1.0, v_0 = 0.3, \delta t = 0.001$,

Plot the trajectories and energy errors in each case as functions of time for $0 \leq t \leq 100$. How do you account for the differences between the graphs?

The problem of course is that the time step, which is fine at $r \approx 1$, begins to fail as the orbital separation r decreases. One way to remedy this is to allow the time step to vary in a manner dictated by the physics. Repeat the previous calculations, but this time choose the time step δt at the start of each step according to the rule

$$\delta t = 0.01 r^{3/2} .$$

Note that the scaling is the same as Kepler's third law. The improvement in accuracy should be obvious. What happens to the orbit's precession and energy conservation?