PHYS 105

In-class exercise 10.1

Orbital Motion – Kepler's Laws

Now let's consider motion entirely above Earth's surface—an orbiting satellite, say, or a planet orbiting the Sun. Modify the simple program (`orbit0.cc/orbit0.py`) on the course web page, with Earth's center at the origin of coordinates and units such that $GM = 1$, to verify the basic laws of orbital motion. Take the time step to be $\delta t = 0.005$. Start the satellite's trajectory at $\mathbf{r} = (r_0, 0)$ with $\mathbf{v} = (0, v_0)$. In the adopted units, a circular orbit should have $v_0 = 1/\sqrt{r_0}$.

(a) Theory predicts that the orbit should be an ellipse with the gravitating object at one focus (Kepler's first law). The conventional measure of the size of an ellipse is the *semi-major axis* a, which is just the average of the satellite's minimum (r_{min}) and maximum (r_{max}) distances from Earth's center: $a = \frac{1}{2}(r_{min} + r_{max})$. Modify your program to follow the motion for one orbital period, operationally defined as the time required for the satellite to return to the positive x-axis after one loop around the origin, and determine numerically the semi-major axes for orbits having

  (a) $r_0 = 1.0, v_0 = 1.0$,
  (b) $r_0 = 1.0, v_0 = 0.9$,
  (c) $r_0 = 1.0, v_0 = 0.7$,
  (d) $r_0 = 1.0, v_0 = 1.1$,
  (e) $r_0 = 1.0, v_0 = 1.3$,
  (f) $r_0 = 2.0, v_0 = 0.6$,
  (g) $r_0 = 2.0, v_0 = 0.9$.

  Show all 7 orbits on *a single plot.*

(b) For the same orbits, determine the orbital period P (as defined above) and compute the ratio P^2/a^3. Are your results consistent with Kepler's third law of planetary motion, which states that this ratio is constant?