Driving the Nonlinear Pendulum

Now let's imagine that in order to overcome the effects of dissipation, our pendulum is also driven by some external force. Think of pushing a swing, but imagine that we don't necessarily push in sync with the motion of the swing itself.

In the small-angle limit $\theta \ll 1$, the pendulum undergoes simple harmonic motion and the natural frequence (without damping) is $\sqrt{k}$. As a simple model, we'll assume that we are pushing with a periodic force but with some other frequency ω_D and amplitude g, so the external acceleration is

$$a_{ext} = g\cos(\omega_D t)$$

and the new equation of motion is

$$\ddot{\theta} = -k\sin\theta - \alpha\dot{\theta} + g\cos(\omega_D t)\,.$$