Dissipation in the Nonlinear Pendulum

No motion is completely free of dissipation. Dissipative forces—friction, air resistance, etc.—oppose motion and do negative work on a particle, tending to reduce the total energy in a non-conservative way. As a general rule, we can take the dissipative force to be of the form

$$\mathbf{F}_{diss} = -f(v)\,\hat{\mathbf{v}},$$

where $f > 0$ and $\hat{\mathbf{v}}$ is a unit vector pointing in the direction of the velocity.

In the case of the pendulum, we might model dissipation in a number of ways. Simple friction has f independent of v, while air resistance may have $f \propto v$ or $f \propto v^2$, as we will discuss later. Here we will take one of the simplest expressions and write $f(v) = Av$, so the equation of motion for the pendulum is

$$mL\ddot{\theta} = -mg\sin\theta - AL\dot{\theta},$$

where we have written $v = L\omega = L\dot{\theta}$. Simplifying and setting $k = g/L$ as before, we find

$$\ddot{\theta} = -k\sin\theta - \alpha\dot{\theta},$$

where

$$\alpha = \frac{A}{m}.$$