Motion with Air Resistance

The dissipative force in air or other fluids can generally be expressed in the form:

$$f_{diss} = \alpha\ v + \beta\ v^2 \,.$$

Either term on the right-hand side of this equation may be important, depending on circumstances.

The first term is most likely to be important in low-speed, *streamline flows*—typically where a reasonably aerodynamic object moves through a fluid at a speed low compared to the speed of sound. The coefficient α depends on the detailed size and shape of the object and on a property of the fluid called its viscosity, but it has no dependence on v. Viscosity, loosely speaking, is the tendency of a fluid to resist shearing stresses. It is often denoted by the Greek letter η (eta). In a few simple cases, the value of α can be computed analytically. For example, in the case of a sphere of radius r moving slowly through a fluid, it can be shown that

$$f_{diss} = 6\pi r\eta v \,.$$

Usually, however, no simple formula exists, and α must be determined experimentally.

The second term is more appropriate to non-streamline and/or high-speed flows. In this case, one can derive the general form of the resistance as follows. Imagine a flat sheet of area A moving through a fluid of density ρ with velocity v perpendicular to the sheet. In time δt the sheet moves through a distance $v\delta t$, and hence sweeps out a volume $Av\delta t$. The mass of fluid in this volume is thus $Av\rho\delta t$. If we further imagine that all of this fluid is accelerated to velocity v (so none "escapes" around the side), the total momentum given to the fluid is $Av\rho\delta tv$. This momentum comes from the moving sheet. The drag force on the sheet is simply its rate of change of momentum, or

$$f_{diss} = -(Av^2\rho\delta t)/\delta t = -A\rho v^2 \,.$$

In practice, this analysis is oversimplified, but it does give the correct functional dependence of f_{diss} on A, ρ and v. More generally, we conventionally define the *drag coefficient* C_d by

$$f_{diss} = -\tfrac{1}{2}A\rho v^2 C_d \,.$$

The dimensionless coefficient C_d is a measure of how "aerodynamic" a body is. It generally has to be determined experimentally. Typical numbers range from 0.1 to over 1.