REVIEW FOR MIDTERM I

Simple Harmonic Motion

$F = -kx$ $\quad x(t) = A\cos(\omega t + \phi)$ $\quad \omega = \sqrt{\frac{k}{m}}$

$f = \frac{1}{T} = \frac{\omega}{2\pi}$ $\quad v(t) = -\omega A\sin(\omega t + \phi)$ $\quad a(t) = -\omega^2 A\cos(\omega t + \phi)$

$k = \frac{mg}{\Delta l}$, for a hanging spring

$A = \sqrt{x_0^2 + \left(\frac{v_0}{\omega}\right)^2}$, $\quad \phi = \cos^{-1}\left(\frac{x_0}{A}\right)$ and $\sin^{-1}\left(-\frac{v_0}{\omega A}\right)$

$$E = K + U = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$$

$\omega = \sqrt{\frac{g}{l}}$ $\quad \omega = \sqrt{\frac{mgd}{I}}$

$x(t) = \left[Ae^{-\frac{b}{2m}t}\right]\cos(\omega' t + \phi)$ where $\omega' = \sqrt{\omega^2 - \left(\frac{b}{2m}\right)^2}$

Mechanical Waves

Traveling wave on a string: $y(x,t) = A\cos(kx - \omega t)$, $\quad k = \frac{2\pi}{\lambda}$, $\quad \omega = \frac{2\pi}{T} = 2\pi f$

$v = \frac{\lambda}{T} = \lambda f = \frac{\omega}{k}$, $\quad v = \sqrt{\frac{F}{\mu}}$, where $\mu = \frac{m}{l}$

$v_y = \frac{\partial y}{\partial t} = \omega A\sin(kx - \omega t)$, $\quad a_y = \frac{\partial^2 y}{\partial t^2} = -\omega^2 A\cos(kx - \omega t)$

Power: $P_{av} = \frac{1}{2}\sqrt{\mu F}\,\omega^2 A^2 = \frac{1}{2}\mu\omega^2 A v$

Intensity $I = \frac{P}{4\pi r^2}$

Superposition Principle

$$y(x,t) = y_1 + y_2 = A\cos(kx - \omega t) + A\cos(kx - \omega t + \phi) = [2A\cos\frac{\phi}{2}]\sin(kx - \omega t + \frac{\phi}{2})$$

Normal modes:

$$y(x,t) = y_1 + y_2 = A\cos(kx - \omega t) + A\cos(kx + \omega t) = [2A\sin kx]\sin\omega t$$

Amplitude maximum or antinode when

, $\sin kx = \pm 1, \Rightarrow kx = \frac{2n+1}{2}\pi$, $\quad x = \frac{2n+1}{4}\lambda$, $\quad$ n=0,1,2, …

Amplitude=0 or node when

$\sin kx = 0, \Rightarrow kx = n\pi$, $\quad x = \frac{n}{2}\lambda$

Separation between two successive nodes $=\lambda/2$

Standing waves on a stretched string fixed at both ends

$y(0,t) = y(L,t) = 0$

$\Rightarrow 2A\sin kL = 0,$

$\Rightarrow kL = n\pi,$ Allowed frequencies: $f_n = \frac{v}{\lambda_n} = \frac{n}{2L}\sqrt{\frac{T}{\mu}} = nf_1$

$\Rightarrow \lambda_n = \frac{2L}{n}$

Electromagnetic waves

Propagation of oscillation of electric and magnetic fields

$\vec{E} = \vec{E}_{\max}\cos(kx - \omega t)$, $\vec{B} = \vec{B}_{\max}\cos(kx - \omega t)$, $E = cB$, $c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$

$n = \frac{c}{v} \approx \sqrt{K}$, $\lambda = \frac{\lambda_0}{n}$

Energy density in em wave: $u = \varepsilon_0 E^2$

EM energy flow per unit cross sectional area per unit time:

Poynting vector $\vec{S} = \frac{\vec{E}x\vec{B}}{\mu_0}$, $S_{av} = I = \frac{E_{\max}B_{\max}}{2\mu_0} = \frac{E_{\max}^2}{2\mu_0 c} = \frac{cB_{\max}^2}{2\mu_0}$

Radiation pressure $P = \frac{I}{c}$ (complete absorption), $P = \frac{2I}{c}$ (total reflection)

Standing waves similar to standing waves on a stretched string.

Nature of Light:

Laws of reflection and refraction: $\theta_r = \theta_a$, $n_a \sin\theta_a = n_b \sin\theta_b$

Total internal reflection: $\sin\theta_{crit} = \frac{n_b}{n_a}$

Dispersion, Rainbows

Polarization: $I = \frac{1}{2} I_0$ for unpolarized wave passing through one polarizing sheet.

$I = I_{\max}\cos^2\phi$ for plane polarized wave passing though one polarizing sheet. ϕ is the angle between the direction of polarization of the incident wave and the transmission axis of the polarizing sheet.

Brewster's angle: $\tan\theta_p = \frac{n_b}{n_a}$