Lecture 15

Midterm 2

Tuesday, August 17
8:00 – 8:50
Disque 103

Chapters: 35, 36, 37, 38

Important equations without any explanation will be provided.

IMPORTANT
Final Exam

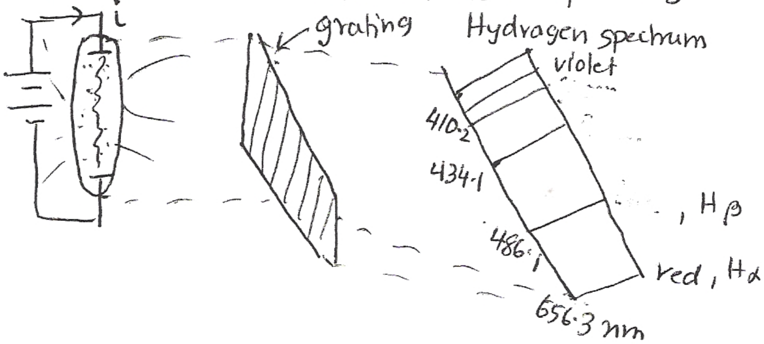
Monday, August 30

~~3~~4:00 PM – ~~5~~6:00 PM

Disque 108

Atomic Spectra of Atoms

If an evacuated glass tube is filled with hydrogen (or some other gas) and a voltage is applied between two metal electrodes in the tube to produce a current, the tube emits light with colors characteristic of the gas.



When the emitted light is analyzed with a spectroscope like a diffraction grating, a series of spectral (discrete) lines is observed, each line correspond to a different color or wavelength. The figure shows the emission spectrum of Hydrogen. Johann Balmer showed in 1885 that the wavelengths produced by hydrogen in the visible region can be described by the following formula

$$\frac{1}{\lambda} = R_H\left(\frac{1}{2^2} - \frac{1}{n^2}\right) \quad \text{where } n = 3, 4, 5 \ldots$$

R is called the Rydberg constant and is given by

$R_H = 1.096776 \times 10^7\ m^{-1}$.

Example: $n = 3$, $\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) \Rightarrow \lambda = 656.3\ nm$, Called H_α

$n = 4$, $\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{4^2}\right) \Rightarrow \lambda = 486.1\ nm$ H_β

$n \to \infty$ λ shortest

This Balmer formula was found to be a special case of a more general expression found by Rydberg and Ritz given by $\frac{1}{\lambda} = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right) \quad n > m$

where n and m are integers and R is called the Rydberg constant which vary only slightly from element to element. For very heavy element $R_\infty = 1.097373 \times 10^7\ m^{-1}$

Other spectral series for hydrogen:

Lyman series: $\frac{1}{\lambda} = R_H\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$, $n = 2, 3 \cdots$

This spectral series lies in the ultraviolet

Paschen series: $\frac{1}{\lambda} = R_H\left(\frac{1}{3^2} - \frac{1}{n^2}\right)$, $n = 4, 5, 6,$ infrared

Brackett series $\frac{1}{\lambda} = R_H\left(\frac{1}{4^2} - \frac{1}{n^2}\right)$, $n = 5, 6, 7$ IR

These wavelengths were also found though not in the visible.

Although the above empirical formula explained the atomic line spectra of not only hydrogen atoms but other elements, at the time there was no theoretical basis for the formula. The question was what atomic structure would give these line spectra both in emission and absorption.

Atomic Models

Many attempts were made to construct a model of the atom that yields the above formula for its radiation spectrum. It was known that an atom was about 10^{-10} m in diameter, that it contained very light electrons and that it was electrically neutral.

Thomson Model of Atom

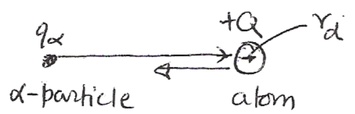
The most popular model of an atom was that of J. J. Thomson who considered the atom to be a fluid of most mass and sufficient positive charge with electrons embedded in it, so that the atom is electrically neutral. Since accelerating or oscillating charge radiates according to electromagnetic theory, it was hoped that normal oscillations of the atom would give the radiation spectrum. But Thomson's model was unable to produce a frequency spectra that would agree with the emission and absorption spectra of atoms.

Rutherford's Model of the Nuclear Atom

Rutherford used α-particles ($^4He^{++}$), a radiation product of radioactive uranium, to study the nature of atoms. He bombarded gold foils with α-particles and expected that the total scattering of the α-particles would be small according to the Thomson model of the atom.

SEE FIGURE, NEXT PAGE

It was found some of the α-particles were scattered by more than 90^0. Some of the α-particles would be scattered by 180^0 when the collision is head on.



By applying conservation of energy, one can get a measure of the radius of the ~~atom~~ positive charge of the atom.

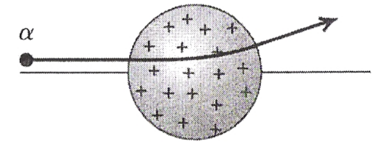
$$E_i = E_f \Rightarrow E_i = \frac{1}{2} m_\alpha v_\alpha^2 = k\frac{q_\alpha Q}{r_\alpha} \Rightarrow r_\alpha = \frac{\frac{1}{2} m v_\alpha^2}{k q_\alpha Q} \sim 10^{-15} m$$

From is extensive experiments Rutherford predicted that the positive charge in an atom is confined in a very small volume ($10^{-15} m$ diameter) along with most of the mass. Electrons move around the nucleus (positive charge) like planets around the sun, bound by the electrostatic attraction of the nucleus.

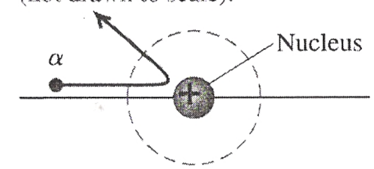
The nuclear atom

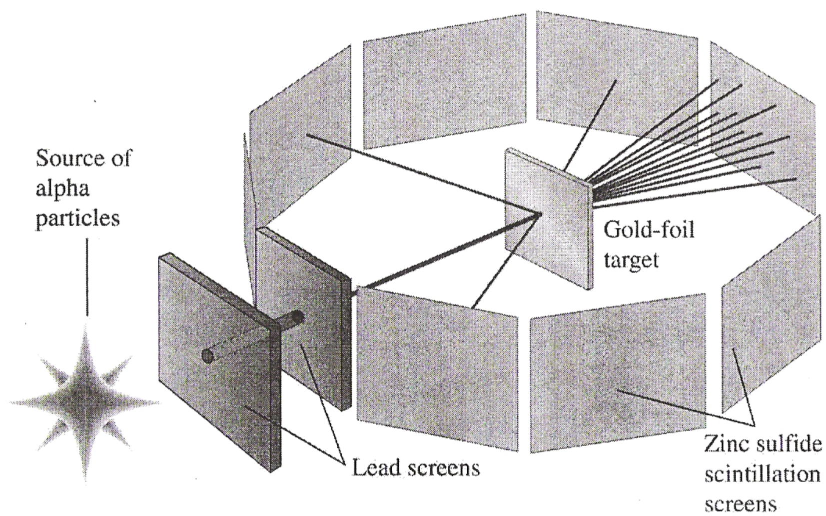
- Rutherford did a very clever experiment with thin Au foil and alpha particles. Scattering was nearly absent or very dramatic, leading him to conclude that the atom was mostly empty space around a dense (+) center.
- Refer to Figure 38.15 (below) and 38.16 (right) and then follow Example 38.5.

(a) Thomson's model of the atom: An alpha particle is scattered through only a small angle.

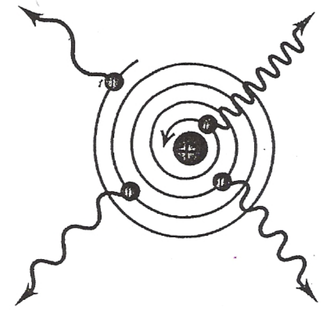


(b) Rutherford's model of the atom: An alpha particle can be scattered through a large angle by the compact, positively charged nucleus (not drawn to scale).





Although such a model of an atom is mechanically stable, it is electrically unstable. The electron making orbits around the nucleus has a centripetal acceleration and thus radiate. Since it loses energy as it radiates, its radius will shrink and eventually it will fall on to the nucleus as shown. Time required for such a collapse will be less than a microsecond. Size of the atom will be the size of the nucleus. But no such thing actually happens.



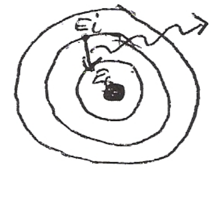
To explain the electrical stability of an atom and calulate the ~~absorption~~ emission and absorption spectra of an atom, Bohr presented a model of the atom which produced results that would agree with experiments.

Bohr's Theory of Hydrogen Atom

The basic assumptions or postulates of the Bohr model as it applies to hydrogen atom as as follows:

1. Electrons can move around the nucleus in certain orbits without radiating. He called these orbits as stationary states.

2. Atoms radiate when an electron makes a transition from from one stationary state of higher energy to another state of lower energy. The frequency of the radiation is related to the energies of the electrons in the orbits by



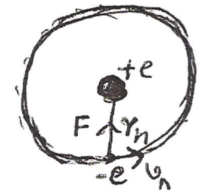
$$hf = E_i - E_f$$

where h is the Planck's constant introduced before.

3. The angular momenta of the electrons in the stationary states are quantized $L = n\hbar$ where $n = 1, 2, 3 \ldots$ and where $\hbar = \frac{h}{2\pi}$.

For circular orbits $\vec{L} = \vec{r} \times \vec{p} = m\, \vec{r} \times \vec{v}$

Thus $L = mvr = n\hbar$ (1)



Applying Newton's second law to circular motion of the electron

$$F = \frac{e^2}{4\pi\epsilon_0 r^2} = m\frac{v^2}{r} \quad (2)$$

Solving for v from Eqn (1) $v_n = \frac{n\hbar}{mr}$ and substituting in Eq. (2)

$$\frac{e^2}{4\pi\epsilon_0 r_n} = m v_n^2 = m\frac{n^2\hbar^2}{m^2 r_n^2}$$

$$\Rightarrow \quad r_n \equiv r = \epsilon_0 \frac{n^2\hbar^2 (4\pi)}{m e^2} = \epsilon_0 \frac{n^2 h^2}{\pi m e^2} \quad (3)$$

This shows that the allowed radii have discrete values or are quantized. $n = 1$ has the smallest radius and is known as Bohr radius a_0.

$$a_0 = r_1 = \epsilon_0 \frac{h^2}{\pi m e^2} = 0.529 \text{ nm}$$

The other radii from Eq. (3) $\boxed{r_n = n^2 a_0}$

The quantization of orbits leads to quantization of the energy of the electrons in the orbits. Total energy of the electron

$$E = K + U = \frac{1}{2}mv^2 - \frac{1}{4\pi\epsilon_0}\frac{e^2}{r}$$

$$= -\frac{1}{2}\frac{e^2}{4\pi\epsilon_0 r}$$

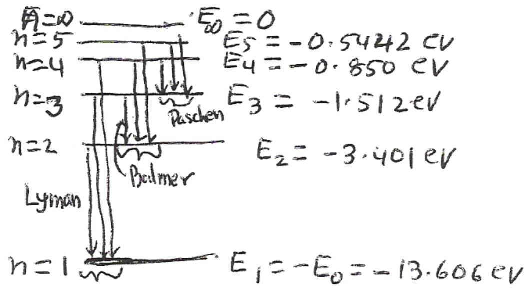
$$\Rightarrow \quad E_n = -\frac{e^2}{8\pi\epsilon_0 r_n} = -\frac{e^2}{8\pi\epsilon_0 a_0 n^2} = -\frac{1}{\epsilon_0^2}\frac{me^4}{8n^2h^2} = -\frac{E_0}{n^2}$$

where $E_0 = \frac{me^4}{\epsilon_0^2 8h^2} = 13.606 \text{ eV}$

Note the lowest allowed energy of the hydrogen atom is $E_1 = -E_0 = -13.606$ eV. The magnitude of this energy is called the ionization energy, because this is the amount of energy that must be supplied to ionize a hydrogen atom from its ground state.

Allowed energy levels are obtained by substituting $n = 1, 2, 3 \ldots$

The energy level diagram of hydrogen atom



$n=1$ with energy $E_1 = E_0 = -13.606$ eV is called the ground state, state with $n=2$, $E_2 = -3.401$ eV is called the first excited state and so on for higher n's.

(See Figure next page)

Radiation from hydrogen atom

An atom radiates when an electron makes a transition from an upper level to a lower level. The frequency of the radiation is obtained from

$$f = \frac{E_i - E_f}{h} = \frac{e^2}{8\pi\epsilon_0 a_0 h}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

In terms of wave length

$$\frac{1}{\lambda} = \frac{f}{c} = \frac{e^2}{8\pi\epsilon_0 a_0 h c}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$= R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

Where $R = \dfrac{e^2}{8\pi\epsilon_0 a_0 h c} = \dfrac{me^4}{8\epsilon_0^2 h^3 c} = 1.097 \times 10^{7}/m$

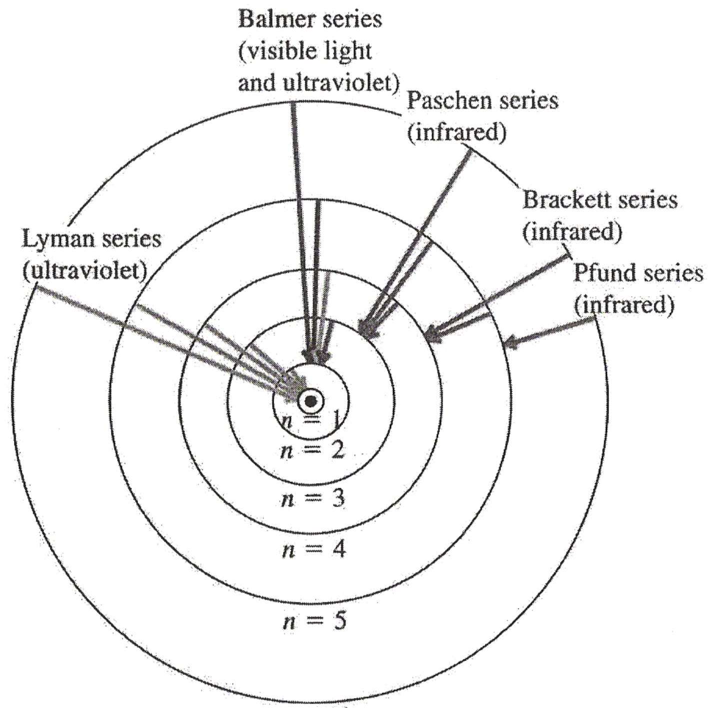
is ~~the~~ Bohr's prediction of Rydberg constant.

Note that this theoretical expression for $\frac{1}{\lambda}$ is identical to the empirical Rydberg equation, ~~provided~~ since R given by the above equation is equal to the Rydberg constant to four significant figures obtained from wavelength measurements. This agreement provides very strong and direct confiration of Bohr's theory.

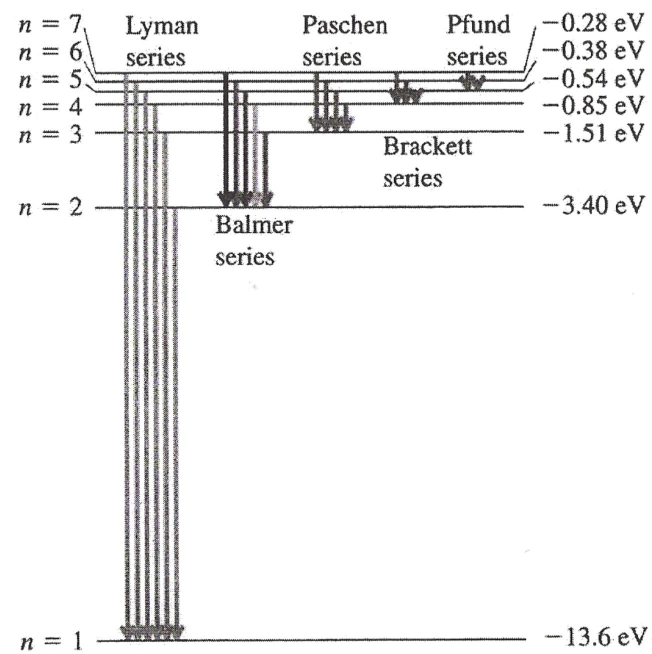
6'

207. Figure 38.9 Two ways to represent the energy levels of the hydrogen atom and transitions between them

(a) "Permitted" orbits of an electron in the Bohr model of a hydrogen atom (not to scale). Arrows indicate the transitions responsible for some of the lines of various series.



(b) Energy-level diagram for hydrogen, showing some transitions corresponding to the various series



Young and Freedman,
University Physics, 12th Edition

The series of wavelengths corresponding to a transition $n_f = 1$, $n_i = 2, 3, 4 \cdots$ as shown in the figure (previous page) is called the Lyman series and they are in the ultraviolet. The series of transition corresponding to $n_f = 2$, $n_i = 3, 4, 5 \cdots$ is called the Balmer series and the wavelengths are in the visible. The series corresponding to $n_f = 3$; $n_i = 4, 5, 6 \cdots$ is the Paschen series and the wavelengths are in the infrared.

$n_f = 4$, $n_i = 5, 6, 7 \cdots$ Brackett series, infrared.

$n_f = 5$, $n_i = 6, 7, 8 \cdots$ Pflund series, infrared.

Example: If $n_f = 1$ and $n_i = 2$, what is the wavelength emitted:

$$\frac{1}{\lambda} = R_H \left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3}{4} R_H$$

(a) $$\lambda = \frac{4}{3R_H} = \frac{4}{3(1.097 \times 10^7 m^{-1})} = 1.215 \times 10^{-7} m = 121.5\ nm$$

(ultraviolet)

(b) what is the frequency of this radiation

$$f = \frac{c}{\lambda} = \frac{3 \times 10^8 m/s}{(1.215 \times 10^{-7} m)} = 2.47 \times 10^{15}\ Hz$$