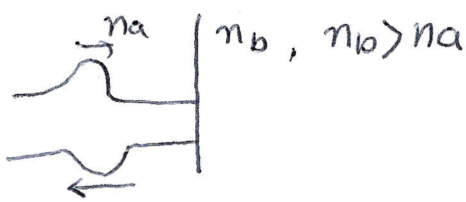
Interference in Thin Films

Appearance of colored bands in oil film on water or in a soap bubble illuminated with white light is the result of interference. Light waves reflected from the front surface and the back surface of the film interfere constructively at different places for different wavelengths and thus we get to see different colors.

Phase change by Reflection

When reflection occurs from the surface of a medium with higher refractive index, the reflected wave undergoes a phase change of π.

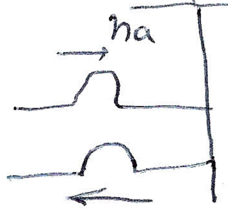


Using Maxwell's equations on reflection and refraction it can be shown that for normal incidence $E_r = \frac{n_a - n_b}{n_a + n_b} E_i$

$\Rightarrow E_r$ + E_i have opposite sign when $n_b > n_a$

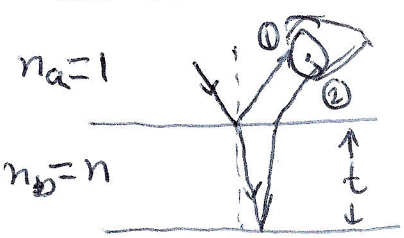
same sign when $n_b < n_a$

If the medium beyond the interface has lower refractive index, there is no phase change in the reflected wave



[See Figure on Page 4]

Near Normal incidence on a thin film



Path difference between two rays $\delta = 2t$

The phase difference between two rays

$$\phi = k\,(\text{path difference } \delta) - \pi$$

$$= 2kt - \pi \quad , \quad k = \frac{2\pi}{\lambda_n}, \; \lambda_n = \frac{\lambda_0}{n}$$

Constructive Interference

$$\phi = 2m\pi \Rightarrow 2kt - \pi = 2m\pi \quad , \quad m = 0, 1, 2 \ldots$$

$$2kt = (2m+1)\pi \Rightarrow \frac{4\pi}{\lambda_n} t = (2m+1)\pi \Rightarrow 4\frac{nt}{\lambda_0} = (2m+1)$$

$$\boxed{2t = (m+\tfrac{1}{2})\lambda_n}$$

$$\boxed{2nt = (m+\tfrac{1}{2})\lambda_0} \quad m = 0, 1, 2 \ldots$$

If the two reflected waves have a multiple of $\frac{1}{2}$ wavelength path difference in the medium, we get a constructive interference.

Destructive Interference

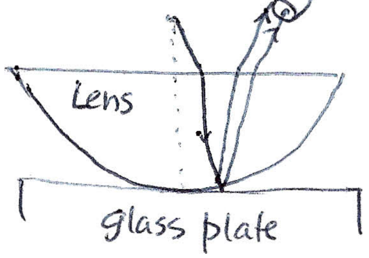
$\phi = 2kt - \pi = (2m+1)\pi$

$2t = (m+1)\lambda_n \Rightarrow 2tn = (m+1)\lambda_0$ with $m = 0, 1, 2 \cdots$

The path difference in the medium must be a multiple of λ in the medium for destructive interference. *

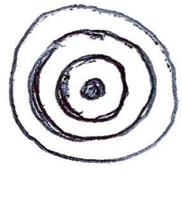
See example in the following page

Newton's Rings



Convex surface of a lens is placed in contact with a plane glass plate and a thin film of air is formed between the two surfaces.

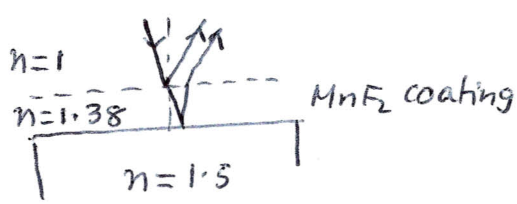
The rays reflected from the curved surface of the lens propagate along with those reflected from the plane surface. They are brought together by the lens of the eye or by an optical instrument. A series of interference rings are observed whenever the proper conditions for constructive and destructive interference are satisfied.



In the reflected light the central fringe is dark because of the extra phase change of π by reflection from the lower surface.

In the transmitted pattern the central fringe is bright.

Nonreflective Coating of glass

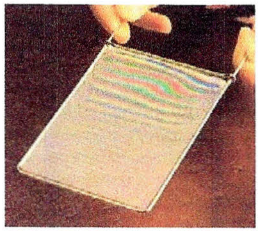


To minimize reflection from ~~lenses~~ or glasses, they are coated with a transparent material (MnF_2, $n=1.38$) of smaller refractive index. If no reflection is intended for $\lambda = 550$ nm (center of visible, yellow-green), what must be the thickness t of the transparent material?

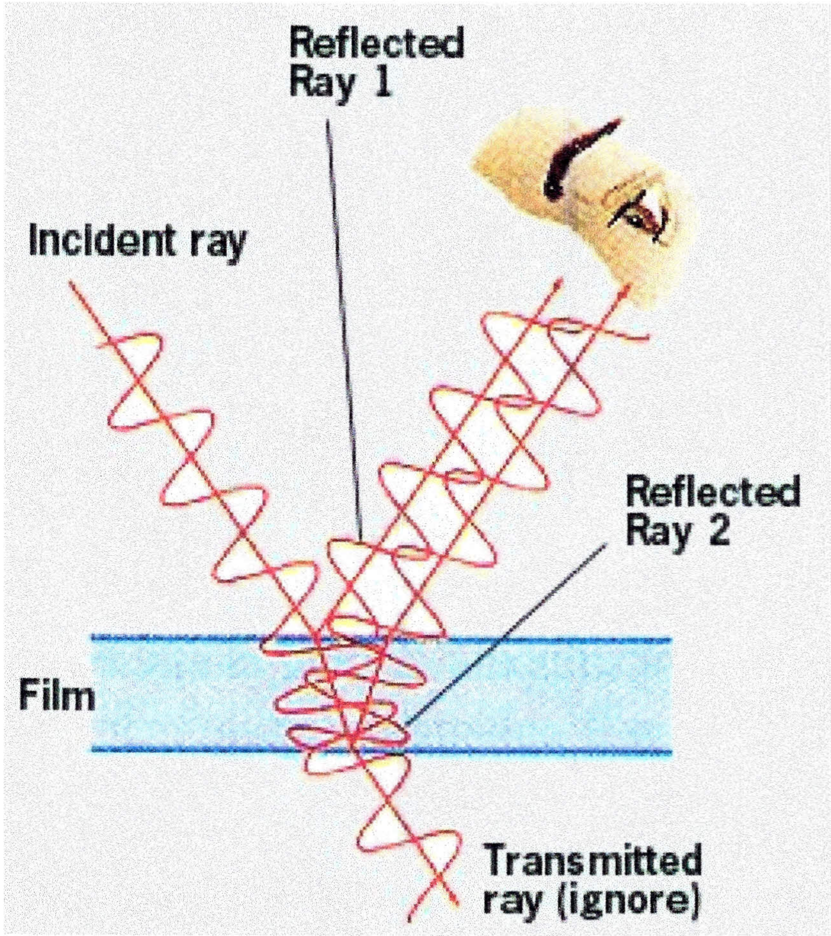
Since phase change occurs at both reflection, for minimum

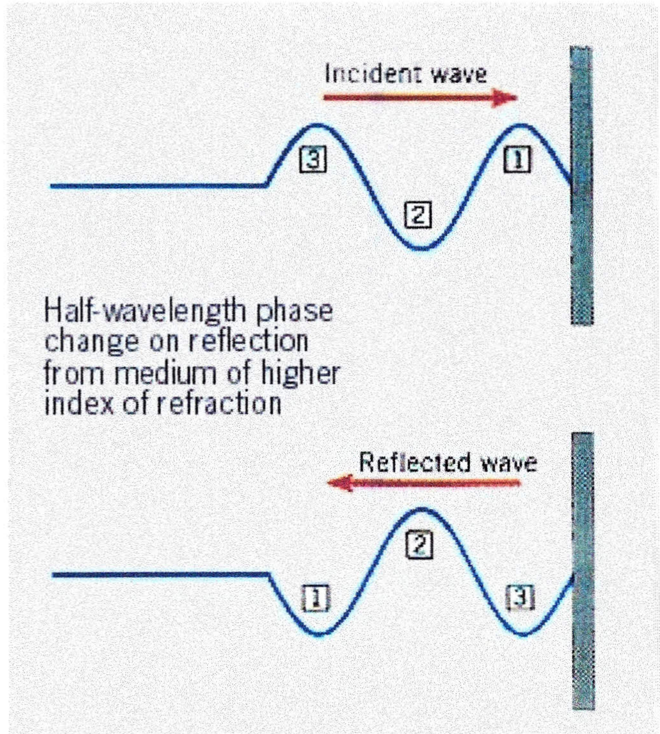
$2d = (m+\frac{1}{2})\lambda_n \Rightarrow d_{min} = \frac{1}{4}\lambda_n = \frac{1}{4}\frac{\lambda_0}{n} = \frac{550}{4(1.38)} \sim 100\,nm$

Note the quarter wavelength thickness of the coating.



Why do we see colors from thin films ?



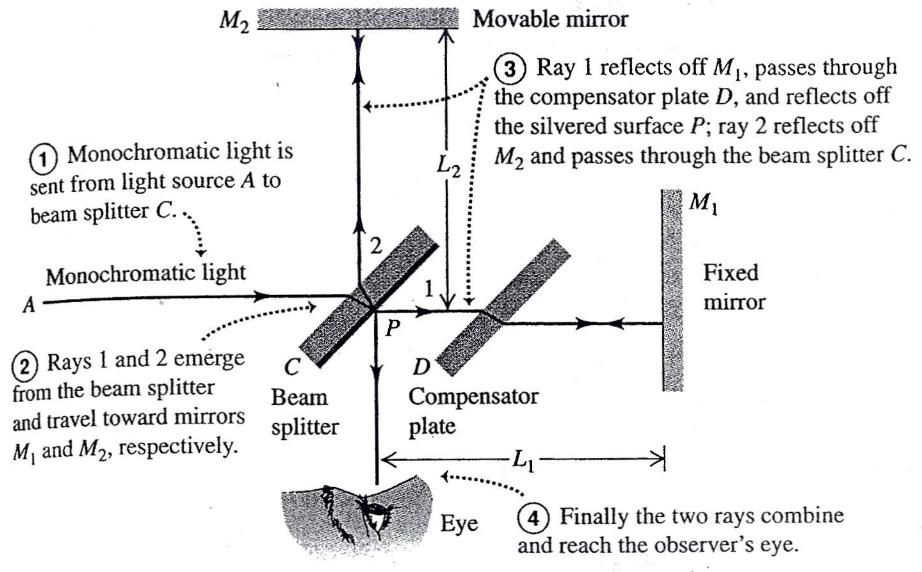


Waves flip when they reflect off a less dense to dense media Interface.

1. White light (which contains all colors) hits a thin film of oil.
2. Some of it is reflected and goes to your eye.
3. Some of it goes into the film and gets reflected by its back side, and goes to eye.
4. How the waves of the various colors add at your eye determines colors that you see.

The Michelson Interferometer

monochromatic source



This is a device that can measure lengths or changes in length with great accuracy by means of interference fringes. It can also be used measure wavelengths by measuring shift in fringes.

Part of the incident monochromatic light passes through the beam splitter (a half silvered mirror) and gets reflected from the fixed mirror M_1. The other part gets reflected at the beam splitter and then gets reflected on the movable mirror M_2. The reflected rays from M_1 and M_2 then reaches the eye by reflection and transmission, respectively. The D is the compensator plate of the same thickness as C, the beam splitter, so that both rays reaching the eye go through the same thickness of material.

The two rays reaching the eye interfere. The interference depends on the path difference of the two rays $L_1 - L_2$.

If M_1 and M_2 are not exactly at right angles, various light rays originating at the source will undergo various path differences when reaching the eye and an interference pattern is formed.

If M_2 is moved through $\frac{\lambda}{2}$, a path difference of λ is introduced and the interference fringes move through one.

If we observe fringe positions through a telescope with a crosshair eye-piece and find that m fringes cross the crosshairs when we move the mirror a distance y, then

$$y = m \frac{\lambda}{2} \quad \text{or} \quad \lambda = \frac{2y}{m}$$

For a large m (several thousands), y can be measured accurately and m can be counted exactly. Then from the above equation λ can be measured accurately.

A change in the fringe pattern can also be caused by insertion of a thin transparent material in the optical path of one of the mirrors - say M_1. This will cause the fringes to shift. By counting the number of fringe shift, we can determine the thickness L of the material. Michelson was able to show using his instrument that the standard meter was equivalent to 1553163.5 wavelengths of red light emitted from a light source containing Cadmium. For this careful measurement Michelson received Nobel Prize in physics in 1907.

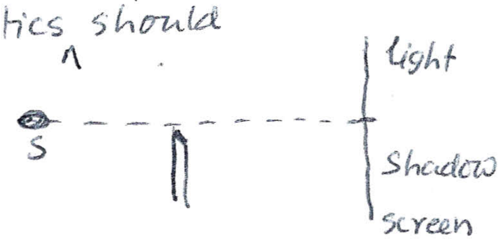
One of the most important applications of the interferometer was the historic Michelson-Morley experiment carried out in 1887 much before the advent of special theory of relativity. They wanted to study the motion of the earth through the ether, the medium in which propagation of light was believed to occur. Their experiment gave a null result i.e. the motion of earth through the ether would not change fringes in the interferometer. This negative result baffled physicist until 1905 when Einstein in his special theory of relativity postulated that light propagates in vacuum with a speed c relative to all inertial frames, independent of their relative velocities. Thus the ether does not play any role and the concept of ether was abandoned.

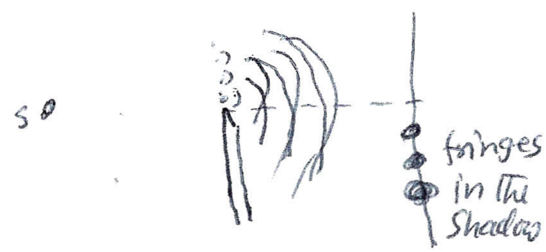
Chapter 36
Diffraction

We know that sound waves can bend around corners (doors and windows). The main reason for this phenomenon which we call diffraction is that sound wave have wavelengths comparable with dimensions of objects which are placed on their path.

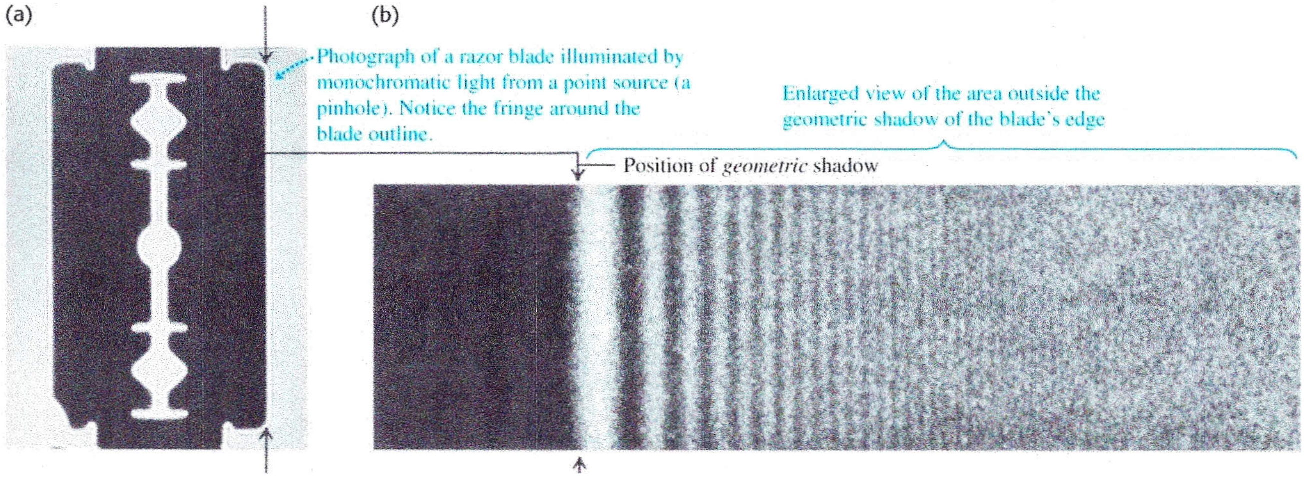
When dealing with visible light the situation is quite different. The visible wavelengths are very small and interference and diffraction effects do not become important unless we deal with slits or apertures of very small dimensions.

Thus although according to ray optics a sharp object should produce a sharp shadow as shown, due to diffraction, light flares into the geometrical shadow of the sharp obstacle. This can be understood on the basis of Huggens's principle that every point on a wave front behaves like a source and produces ~~sph~~ spherical waves in the foward direction as shown in the second figure. This diffraction ~~can~~ reveals fringes on the screen located well beyond the geometrical shadow. See the figure on the next page.

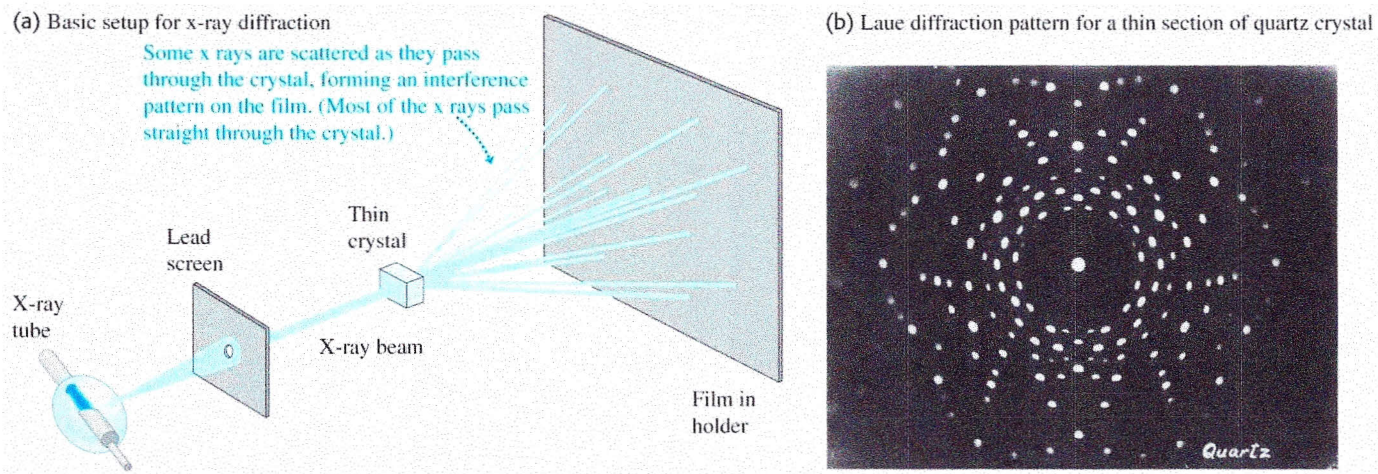




Diffraction: Bending of light around corners

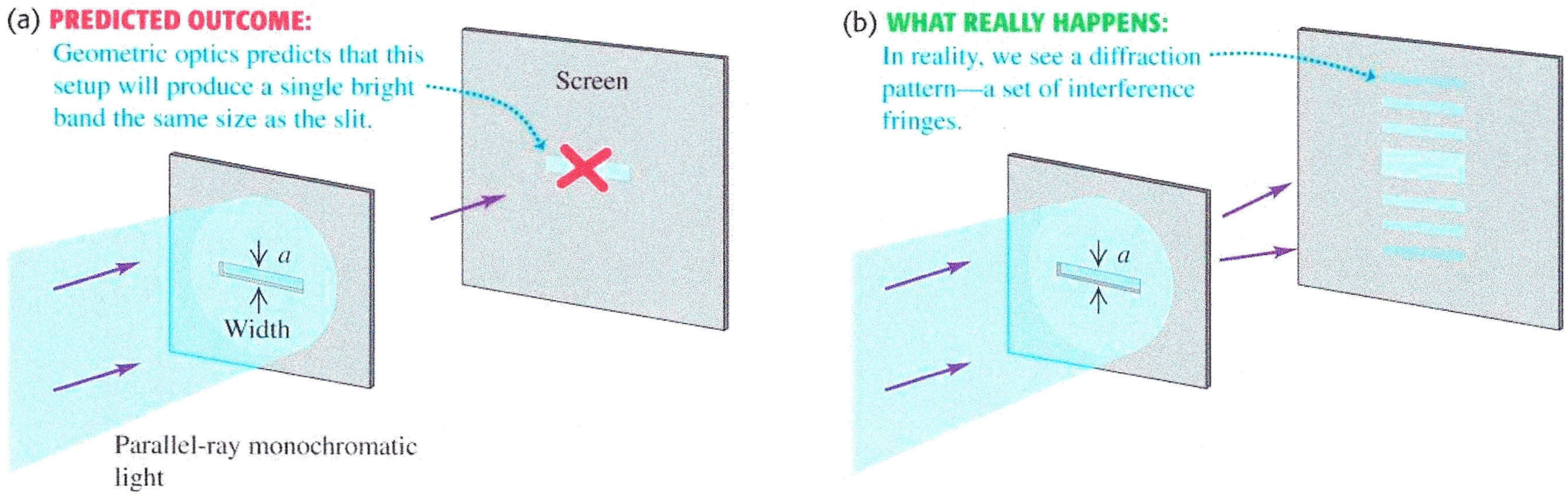


Area behind a geometric shadow is bordered by interference fringes



Diffraction is useful in materials science

Light: Particle or Wave ?



Predictions of geometric and wave optics