Lecture 5

Chapter 32
Electromagnetic Waves

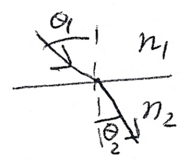
Nature of Light

1. Corpuscular or Particle Nature – Proposed by Newton

Explains
- a) straight line propagation
- b) Law of reflection

$\theta' = \theta$

Does not explain
- a) Law of refraction
- b) Interference, Diffraction



$n_1 \sin\theta_1 = n_2 \sin\theta_2$

2. Wave Nature (work of Huygens, Young, Fresnel, Maxwell)

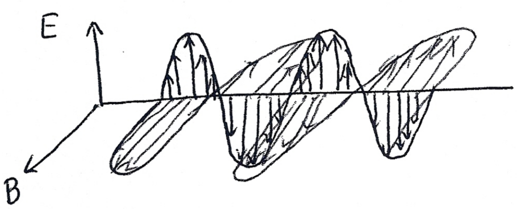
Light is a wave phenomenon, can bend around corners

Explains

Reflection, refraction, interference, diffraction and polarization

Maxwell's Theory

Light is an electromagnetic wave i.e., propagation of a light wave corresponds to propagation of oscillations of electric and magnetic fields perpendicular to the direction of propagation



Direction of propagation

Maxwell equations (given below) show that a time varying magnetic field acts as a source of electric field and vice versa. These $\vec{E}$ and $\vec{B}$ fields sustain each other, forming an electromagnetic wave that propagates through space.

Waves produced by sources at TV and radio stations, microwave oscillators for ovens and radar, x-ray machines and radio active nuclei are all electromagnetic waves.

Unlike mechanical waves em-waves can propagate through vacuum, (eg. starlight)

Maxwell's Equations and Electromagnetic Waves

We have seen before (Physics 102) that an electric charge an produce an electric field and a moving charge with a constant speed can produce both electric and magnetic fields. But these fields can be treated independently and cannot produce electromagnetic wave. However, an oscillating or accelerating charge produces oscillating electric and magnetic field which sustain each other and propagate through medium as electromagnetic wave. An em-wave is also produced by a time varying electric current.

Maxwell in 1965 discovered that the basic principles of electromagnetic waves in terms of four equations now known as Maxwell's equations. These are

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0} \quad \text{(Gauss's law)}$$

$$\oint \vec{B}\cdot d\vec{A} = 0 \quad \text{(Gauss's law for magnetism)} \Rightarrow \text{no monopole}$$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 i_c + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt} \quad \text{(Ampere's law generalized by Maxwell)}$$

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt} \quad \text{(Faraday's law)}$$

Where the flux is: $\Phi_{B,E} = \int_A \vec{B}\cdot d\vec{A}$ or $\int_A \vec{E}\cdot d\vec{A}$

Combining the differential forms of these equations, Maxwell was able to show that the E-field and the B-field satisfy the wave equation, i.e.

$$\frac{\partial^2 \vec{E}}{\partial x^2} = \epsilon_0 \mu_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

and

$$\frac{\partial^2 \vec{B}}{\partial x^2} = \epsilon_0 \mu_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

cf: The general form of the wave equation derived before

$$\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y(x,t)}{\partial t^2}$$

This shows that if the E-field and B-field satisfy

These wave equation, then the speed of light in vacuum must be idientied as

$$v \equiv c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

Remembering $\epsilon_0 = 8.854 \times 10^{-12} \frac{C^2}{N m^2}$.

and $\mu_0 = 4\pi \times 10^{-7} \frac{T.m}{A}$

We find that $c = 2.997 \times 10^{8}$ m/s which is exactly the measured velocity of light in vacuum.

Generation of Electromagnetic Waves

Electromagnetic waves were first produced in the lab in 1887 by German physicist Heinrich Hertz using charges oscillating in an L-C circuit (Phys 102) as a source. In a radio transmitter, oscillating charges on a long antenna produce the radio waves.

Electromagnetic Spectrum

Electromagnetic wave frequencies span a wide spectrum from 1 Hz to 10^{24} Hz. The whole spectrum consist of radio + TV waves, microwaves, infrared, visible, ultraviolet, x-rays and gamma rays. The wavelength (frequency) range of visible light is 400nm to 700nm (750 to 430 THz or 7.5 to 4.3×10^{14} Hz). See figure 32.4 and Table 32.1 of text.

Plane Electromagnetic Waves

The simplest solution of the two wave equation is

$$\vec{E}(x,t) = \vec{E}_{max} \cos(kx - \omega t)$$

$$\vec{B}(x,t) = \vec{B}_{max} \cos(kx - \omega t)$$

where, as before, $k = \frac{2\pi}{\lambda}$ and $\omega = \frac{2\pi}{T} = 2\pi f$ and $c = \frac{\omega}{k} = \lambda f$.

It follows from Maxwell's theory that $\vec{E}$ is normal to B and both $\vec{E}$ and $\vec{B}$ are normal to the direction of propagation. Then if we choose $\vec{E}_{max} = |E_{max}|\hat{j}$ and $\vec{B}_{max} = |B_{max}|\hat{k}$, Then the wave propagates in the x-direction. Such a solution is called a linearly polarized wave and the above solution is a plane polarized wave.

From Faraday's law of induction it can be shown that

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}, \text{ For the above plane wave soln}$$

$$\frac{\partial E}{\partial x} = -k E_{max} \sin(kx-\omega t) \quad \text{and} \quad \frac{\partial B}{\partial t} = \omega B_{max} \sin(kx-\omega t)$$

$$k E_{max} = \omega B_{max} \Rightarrow \boxed{\frac{E_{max}}{B_{max}} = \frac{\omega}{k} = c}$$

<u>Example</u>: Sinusoidal wave of frequency 40 MHz travels in free space

(a) wavelength = ? $\quad \lambda = \frac{c}{f} = \frac{3\times10^8 \, m/s}{4\times10^7/s} = 7.5 \, m$

(b) Period = ? $\quad T = \frac{1}{f} = \frac{1}{4\times10^7} = 2.5\times10^{-8} \, s$

(c) If $E_{max} = 750 \, N/c \, \hat{j}$, $\vec{B}_{max} = ?$

$$|B_{max}| = \frac{|E_{max}|}{c} = \frac{750 \, N/C}{3\times10^8 m/s} = 2.5\times10^{-6} \, T$$

Since E and B are $\perp$ to each other and each $\perp$ to x, if $\vec{E}$ is in the y-direction, $\vec{B}$ must be in the z-direction.

<u>Electromagnetic Waves in Matter</u>

For em-waves in matter ϵ_0 and μ_0 in Maxwell's equations and wave equations are replaced by ϵ and μ. The permittivity and permeability of the medium. $\epsilon = K\epsilon_0$ and $\mu = K_m \mu_0$ where K and K_m are the dielectric constant and relative magnetic permeability of the medium, In this case the speed of light in the medium is

$$v = \frac{1}{\sqrt{\epsilon\mu}} = \frac{1}{\sqrt{KK_m}} \frac{1}{\sqrt{\epsilon_0\mu_0}} = \frac{c}{\sqrt{KK_m}} \approx \frac{c}{\sqrt{K}} \quad \text{since } K_m \cong 1 \text{ for most substances.}$$

$$\Rightarrow \frac{c}{v} = n = \sqrt{KK_m} = \sqrt{K}, \quad n = \text{refractive index.}$$

Energy and Momentum in Electromagnetic Waves

It is obvious that electromagnetic waves carry energy; energy in sun's radiation, microwave ovens, radio transmitters, lasers, etc.

In the E+M course, you have seen that

the energy density in the electric field is $u_E = \frac{1}{2}\epsilon_0 E^2$

and " " " " magnetic " " $u_B = \frac{1}{2}\frac{B^2}{\mu_0}$

So in the electromagnetic field the total energy density is

$$u = u_E + u_B = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\frac{B^2}{\mu_0}$$

$$= \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0}\left(\frac{E}{c}\right)^2 = \frac{1}{2}\epsilon_0 E^2 + \frac{E^2}{2\mu_0}\mu_0\epsilon_0$$

$$\Rightarrow \boxed{u = \epsilon_0 E^2 = \frac{B^2}{\mu_0}}$$

This energy density is function of position and time.

Electromagnetic Energy Flow and the Poynting Vector

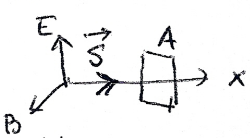
As the electromagnetic wave propagates, the $\vec{E}$ and the $\vec{B}$ fields carry this energy with them. The amount of energy carried per unit time per unit area can then be written as

$$S = uc = \epsilon_0 c E^2 = \epsilon_0 \frac{1}{\sqrt{\mu_0\epsilon_0}} E^2$$

$$\Rightarrow \boxed{S = \sqrt{\frac{\epsilon_0}{\mu_0}}\, E^2 = \frac{EB}{\mu_0}}$$

This rate of flow of energy can be described by a vector called, Poynting vector

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$$



This shows flow of energy per unit time per unit area in the direction of propagation.

The intensity of the electromagnetic wave is defined as the time average of the Poynting vector

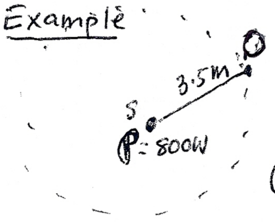
$$I = S_{ave} = \frac{\overline{EB}}{\mu_0} = \frac{E_{max}B_{max}}{\mu_0}\overline{\cos^2(kx-\omega t)}$$

$$\frac{\int_0^T \cos^2(kx-\omega t)}{T} = \frac{1}{2}$$

$$\boxed{I = \frac{E_{max} B_{max}}{2\mu_0} = \frac{E_{max}^2}{2\mu_0 c} = \frac{1}{2}\sqrt{\frac{\epsilon_0}{\mu_0}}\, E_{max}^2 = \frac{1}{2}\epsilon_0 c\, E_{max}^2}$$

This is the intensity of a sinusoidal wave in vacuum.

Example:



(a) Intensity at O, $I = \frac{P}{4\pi r^2} = \frac{800}{4\pi (3.5)^2}$

$= 5.20\ W/m^2$

(b) $E_{max} = ?$ $\quad I = \frac{E_{max}^2}{2\mu_0 c} \Rightarrow E_{max} = \sqrt{2\mu_0 c I}$

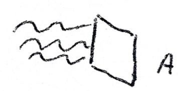
$$E_{max} = \sqrt{2 \times (4\pi \times 10^{-7})(3\times 10^8)(5.20)} = 62.6\ V/m$$

(c) $B_{max} = ?$ $\quad B_{max} = \frac{E_{max}}{c} = \frac{62.6\ V/m}{3\times 10^8\ m/s} = 2.09\times 10^{-7}\ T$

E_{max} is large and B_{max} is small. Most detectors of em wave, therefore, respond to electric field and not magnetic field.

Electromagnetic Momentum Flow and Radiation Pressure

Electromagnetic waves transport energy as well as linear momentum. This means that pressure is exerted on a surface when em-wave impinges on it.



If total energy U of the em-wave is absorbed by the surface, then Maxwell showed that the total momentum $\vec{p}$ delivered to the surface

$$p = \frac{U}{c} \quad (\text{complete absorption})$$

Pressure exerted on the surface

$$P = \frac{F}{A} = \frac{1}{A}\frac{dp}{dt} = \frac{1}{c}\frac{d}{dt}\left(\frac{U}{A}\right) = \frac{1}{c}\left(\frac{1}{A}\frac{dU}{dt}\right)$$

Poynting vector

$$\Rightarrow \quad \boxed{P = \frac{S_{ave}}{c} = \frac{I}{c}} \quad \text{for complete absorption}$$

This is what happens in black body radiation which absorbs all electromagnetic waves incident on it

If the surface is a perfect reflector, the momentum delivered for normal incidence on the surface during reflection of energy U is

$$p = \frac{2U}{c}$$

($\frac{U}{c}$ during incidence and $\frac{U}{c}$ during reflection)

So the total pressure exerted on a perfect reflecting surface is

$$\boxed{P = \frac{2S_{ave}}{c} = \frac{2I}{c}} \quad \text{for perfect reflection}$$

Example: Sun delivers about $1000\ W/m^2$ of energy per second per m^2 to the earth's surface.

a) Power $\mathcal{P} = ?$ on a roof of surface area $8m \times 20m$

$$\mathcal{P} = IA = \left(1000\ \frac{W}{m^2}\right)(8m \times 20m) = 1.6 \times 10^5\ W$$

(only a fraction of this power can be converted to useful purposes)

b) Radiation pressure assuming the roof is a perfect absorber

$$P = \frac{I}{c} = \frac{1000\ W/m^2}{3 \times 10^8\ m/s} = 3.33 \times 10^{-6}\ \frac{N}{m^2}$$

This is a very small pressure, of the order 10^{-10} atmospheric pressure, but it can be measured with sensitive instruments.